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# Soft Prioritized Network Coding in Multiple Access Relay Networks

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**Soft prioritized network coding in multiple access relay networks**

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## DEDICATION

I would like to dedicate this thesis to my parents, my wife, my sister, my brother-in-law, my cousin brother and my sister-in-law without whose support I would not have been able to complete this work.

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## ABSTRACT

We propose soft prioritized network coding techniques that provide a variable quality of service (QoS) to different nodes in multiple access relay networks. The basic idea is to exclude the low priority nodes in the network encoder when their channel gains towards the destination are below a threshold or above a threshold, and vary the threshold to provide a variable QoS. The proposed techniques can provide a soft-level prioritized service to different nodes depending on their assistance needs. They are a natural generalization of current prioritized network coding techniques which can provide only a hard-level (fixed) prioritized service. We derive the probability of decoding error for the prioritized and non-prioritized nodes in Rayleigh flat fading channel with path loss and additive white Gaussian noise taking into account the channel estimation error.

# CHAPTER 1. COOPERATIVE COMMUNICATION

Cooperative communication is a technique by which single antenna mobiles cooperate to create virtual MIMO like system and reap diversity benefits of MIMO systems(1; 2; 3; 4). The independent copies of same signal are decoded at the destination resulting in efficient combating to fading. A simple cooperative system is shown in Fig. 1.1.

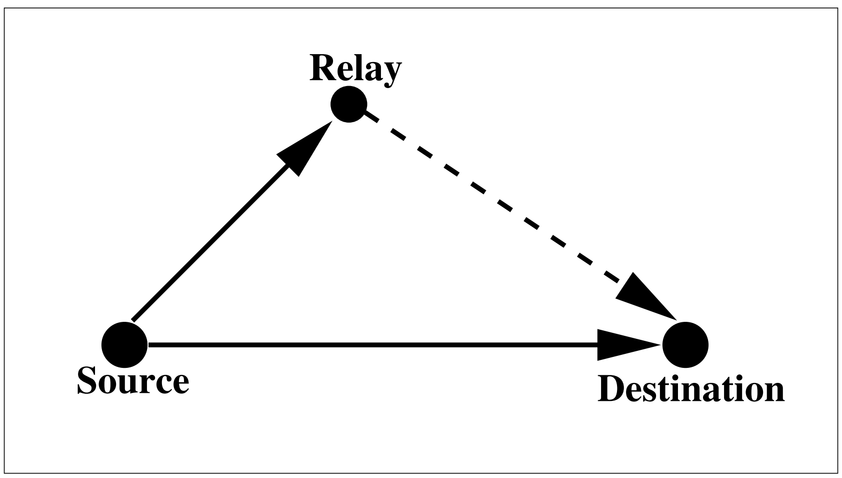


Figure 1.1 A simple cooperative communication system

It consists of a source(S), a relay(R) and a destination(D). In cooperative networks, transmission typically occurs in two phases. In first phase the source sends its message to the destination. Because of the broadcast nature of the wireless medium, the relay overhears the signal transmitted by the source. In second phase the relay assists the source by re-transmitting source's signal according to a certain protocol to destination. Several protocols have been proposed in the literature for relay's operation in second phase. The notable ones are(5; 6)

- **Amplify and Forward (AF)**: In the AF protocol, the relay node amplifies the received signal and forwards the amplified version to the destination. The amplification gain at the relay node is chosen according to the relay power constraint.

- **Decode and Forward (DF)**: In the DF protocol, the relay node decodes the received signal, re-encodes it, and forwards the encoded signal to the destination.
- **Compress and Forward (CAF)**: The CAF is the cooperative protocol which allows the relay node to compress the received signal from the source node and forward it to the destination without decoding the signal. The Wyner-Ziv coding can be used for optimal compression.

The destination combines the signal received from source in first phase and from relay in the second phase to decode the original message of the source.

## 1.1 Cooperative Coding

Cooperative Coding(8; 9) is a method that merges ideas of channel coding and cooperative communication. In this technique, different portions of the source codeword are sent by two independent fading paths. The basic idea is that the relay transmits incremental redundancy to the destination. The destination forms a lower rate code using the messages received from source and relay which improves decoding performance. Typically cooperative coding requires relay to correctly decode source message.

## 1.2 Network coding

Network Coding (7) is a technique in which, instead of simply relaying the packets, the intermediate nodes of a network combine several received packets and transmit them to other nodes. This can be used to attain the maximum possible information flow in a network. It has been proved to achieve multicast capacity of a network. In multiple access relay networks(MARN), multiple sources communicate to a common destination. The intermediate relay nodes perform network coding and transmit the coded packets to the destination. The destination decodes the source message using the packets received from relays and the sources.

# CHAPTER 2. RELATED WORK

In this chapter we will review some of the existing coding techniques proposed for prioritizing user data using network coding. All of them typically assume binary erasure channel model and do not take channel variations of wireless channel into account.

Authors in (10) proposed the use of Stacked Linear Codes (SLC) and Progressive Linear Codes (PLC) as *priority random codes*. *Priority Random Codes* are the codes which are able to recover those subsets of original data which is more important or, in other words, is of higher priority. This is achieved by ensuring that the coded blocks corresponding to high priority data is of linear combination of *fewer* source blocks, as compared to other non-prioritized data.

They assume N source blocks are generated which are classified to $n$ different priority levels, level i being more important than j and each level has $a_i$ blocks. Both Stacked and Progressive are based on Random Linear Codes (RLC). RLC generated coded blocks $c_i$ by combination of all N source blocks as $c_i = \sum_{j=1}^{N} \beta_{i,j} x_j$ where the coefficients $\beta_{i,j}$ are randomly chosen from a Galois field. In SLC, all the source blocks of kth level are combined to generate required number of coded block belonging to kth set ie $c_i = \sum_{j=b_{k-1}+1}^{b_k} \beta_{i,j} x_j$, where $\beta_{i,j}$ is a non-zero random number. In PLC, the source blocks are encoded in descending priority. The kth level block is encoded using blocks between level 1 and k i.e $c_i = \sum_{j=1}^{b_k} \beta_{i,j} x_j$. The decoding for SLC is Gaussian elimination whereas for PLC its Gauss-Jordan elimination.

Authors in (12) proposed scheme in which Global Encoding Kernels (GEK) are optimized to provide unequal error protection to different users. The channel model assumed here is binary erasure channel model. They assume one source multicasting data to different destinations where intermediate nodes can perform network coding. In this scheme first sorting of erasure probability is performed such that $P_{e,ik} \leq P_{e,ir}$ where $P_{e,ij}$ denotes total erasure probability of $j^{th}$ path of the $i^{th}$ sink. Then GEK are assigned progressively such that most of the important data can be recovered using as paths with lower erasure probability. This optimization involves decomposition of network graph into line graphs with different coding operations. After that an exhaustive search over all possible coding operations is performed to assign optimal GEKs. Since this exhaustive search space can become too large, they have suggested a suboptimal iterative algorithm to assign GEKs.

Authors in (13) have proposed a random coding scheme for unequal error protection (UEP). The channel model assumed here is binary erasure channel model. The basic idea behind the scheme is to formulate a optimization problem solved locally by each intermediate node which determines the number of packets of each priority class to be requested from parent nodes. These intermediate nodes are not interested in retrieving original packets, but to forward linear combinations of those packets. The objective function is a log-concave function which can be solved using simple greedy algorithm to determine optimal coding operations. The algorithm can be distributively applied to every nodes in the network and performance improvement for layered multimedia transmission is achieved.

Authors in (11) have proposed use of Rank metric codes or Gabidulin codes for unequal protection. Gabidulin codes are maximum distance codes in rank metric. They are similar to Reed Solomon(RS) Codes in sense that RS codes are maximum distance in Hamming metric. A Gabidulin code of dimension $k$ can be correctly decoded after the reception of any $K \geq k$ linearly independent linear combination of the codeword symbols. The authors propose use of Gabidulin code with parameters $(n, k_i, n - k_i + 1)$ in finite field $\mathbb{F}_{q^m}$, symbol length be $2^w$ and $n = w \cdot \log_q 2$.

# CHAPTER 3. SOFT PRIORITIZED NETWORK CODING

In this chapter the *soft prioritized network coding* technique that provide a variable QoS to different nodes by adapting the network encoding rule to the channel gain between the source and destination node is described. A two-hop multiple access relay network where two source nodes communicate with a common destination with the assistance from a relay is considered. The basic idea is to exclude the non-prioritized node in the network encoder when their channel gains towards the destination is below a threshold (bad enough) or above a threshold (good enough), and vary the thresholds to provide a variable QoS. They are a natural generalization of current prioritized network coding techniques which can provide only a hard-level (fixed) prioritized service.

## 3.1 System Model

The system model in consideration is a multiple access relay network, shown in Fig. 3.1, where the source nodes $S_1$ and $S_2$ send message bits, $m_1, m_2 \in \{+1, -1\}$, using antipodal Binary Phase Shift Key (BPSK) modulation to a common destination D through orthogonal channels (time or frequency). The relay R after overhearing $m_1$ and $m_2$ and decoding them correctly (indicated by CRC check), generates a parity bit $p$ by XORing $m_1$ and $m_2$ and sends the parity bit to D. Let

$$\begin{aligned} y_1 &= h_1 m_1 \sqrt{d_1^{-\alpha} E_s} + n_1 \\ y_2 &= h_2 m_2 \sqrt{d_2^{-\alpha} E_s} + n_2 \\ y_r &= h_r p \sqrt{d_r^{-\alpha} E_r} + n_r \end{aligned} \tag{3.1}$$

denote the received signals at D from $S_1$, $S_2$ and R, respectively, where the channel gains $h_1$, $h_2$ and $h_r$ are the complex Gaussian random variables with mean zero and variance one, $d_i$ is

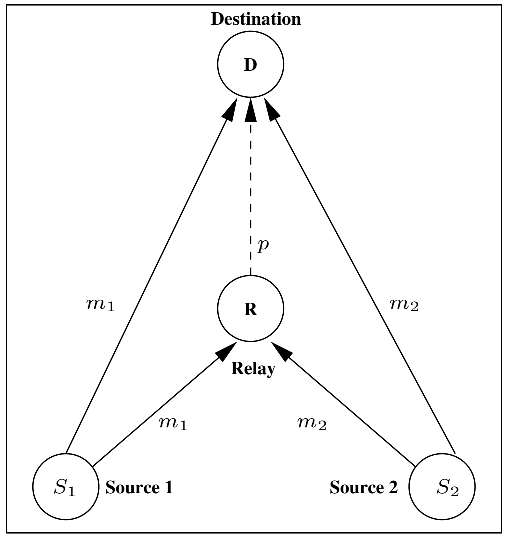


Figure 3.1 System Model

the distance between node $i \in \{1, 2, r\}$ and D; $\alpha$ is the path loss exponent; $n_1$, $n_2$ and $n_r$ are complex white Gaussian noise with mean zero and variance $N_0/2$ per dimension; and $E_s, E_r$ are the transmit symbol energy at the source and relay, respectively.

Considering first the traditional network coding $p = m_1 \oplus m_2$, where 1 the additive identity element under $\oplus$ (modulo-2) addition. The destination decodes the source message bits using the maximum a posteriori (MAP) decoding rule whose reliability of decision is measured by the magnitude of log-likelihood ratio (LLR). The LLR for $m_1$ at D for given $\mathbf{h} = (h_1, h_2, h_r)$ and $\mathbf{y} = (y_1, y_2, y_r)$ is given by

$$\begin{aligned} L(m_1|\mathbf{h}, \mathbf{y}) &= \ln\left(\frac{Pr(m_1 = +1|\mathbf{h}, \mathbf{y})}{Pr(m_1 = -1|\mathbf{h}, \mathbf{y})}\right) \\ &= L(m_1|h_1, y_1) + L(p \oplus m_2|h_r, y_r, h_2, y_2) \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} L(m_1|h_1, y_1) &= \ln\left(\frac{Pr(m_1 = +1|h_1, y_1)}{Pr(m_1 = -1|h_1, y_1)}\right) \\ &= \frac{4\sqrt{d_1^{-\alpha} E_s}}{N_0} Re\{h_1^* y_1\} \end{aligned} \tag{3.3}$$

is the LLR of $m_1$ for given $h_1$ and $y_1$, and

$$L(p \oplus m_2|h_r, y_r, h_2, y_2) = \frac{1 + e^{L(p|h_r,y_r)}e^{L(m_2|h_2,y_2)}}{e^{L(p|h_r,y_r)} + e^{L(m_2|h_2,y_2)}}$$
$$\approx \text{sgn}(L(p|h_r, y_r))\text{sgn}(L(m_2|h_2, y_2)) \min \{|L(p|h_r, y_r)|, |L(m_2|h_2, y_2)|\} \tag{3.4}$$

is the LLR of $m_1$ for given $h_r, h_2, y_r, y_2$. Eq.(3.3) is the information on $m_1$ provided by $S_1$ and Eq.(3.4) shows that the reliability of the additional information on $m_1$ provided by the relay is determined by the least reliable node. The MAP decision rule is to decide $m_1 = 1$ if $L(m_1|\mathbf{h}, \mathbf{y}) > 0$, otherwise, decide $m_1 = -1$. The reliability (confidence) of decision on $m_1$ is determined by the magnitude of $L(m_1|\mathbf{h}, \mathbf{y})$. Similarly, $L(p \oplus m_1|h_r, y_r, h_1, y_1)$ is the additional information on $m_2$ provided by the relay.

### 3.2 Soft Prioritized Network Coding I

From Eq.(3.2) and Eq.(3.4), the magnitude of LLR, $|L(m_1|\mathbf{h}, \mathbf{y})|$, is bounded by:

$$|L(\hat{m}_1|\mathbf{h}, \mathbf{y})| \leq \; |L(m_1|h_1, y_1)| \; + \min\{|L(p|h_r, y_r)|, |L(m_2|h_2, y_2)|\} \tag{3.5}$$

where second term represents the reliability gain for $m_1$ provided by the relay and the reliability gain is determined by the reliability of the weakest link, i.e. $\min\{|L(p|h_r, y_r)|, |L(m_2|h_2, y_2)|\}$. This means if R-to-D or $S_2$-to-D is in deep fading, the reliability gain provided by the relay is very limited. In practice, however, the R-to-D channel may be assumed to be good. [1] Hence, the reliability gain for $m_1$ will be determined by $|L(m_2|h_2, y_2)|$. This observation motivates to propose an adaptive prioritized encoding rule that takes advantage of the channel variations.

Without loss of generality assuming that $S_1$ is the prioritized source. Then, the proposed prioritized network encoding rule is

$$p = \begin{cases} m_1 \oplus m_2 & \text{if } |L(m_2|h_2, y_2)| \geq \beta \\ m_1 & \text{if } |L(m_2|h_2, y_2)| < \beta \end{cases} \tag{3.6}$$

for some threshold $\beta$. This will be referred as LLR-based *Soft Prioritized Network Coding I* (SPNC - I).

[1] Otherwise, the relay would not have been selected to help $S_1$ and $S_2$.

In practice $|L(m_2|h_2, y_2)|$ may not be available instantaneously. It can, however, be approximated by the signal-to-noise ratio (SNR) $g_2 E_s/N_0$, where $g_2 = |h_2|^2 d_2^{-\alpha}$. This leads to a more practical encoding rule

$$p = \begin{cases} m_1 \oplus m_2 & \text{if } g_2 \geq \beta_1 \\ m_1 & \text{if } g_2 < \beta_1 \end{cases} \tag{3.7}$$

for some threshold $\beta_1$. This will be referred to as *SPNC-I*. It should be noted that the traditional network coding scheme $p = m_1 \oplus m_2$ corresponds to the special case of $\beta_1 = 0$. Also, the conventional hard prioritized network coding scheme ($p = m_1$) corresponds to the special case of $\beta_1 = \infty$.

*Generalization*: The SPNC-I scheme can be readily generalized to multi-source scenarios. Suppose $K$ source nodes $S_1, S_2, ..., S_K$ transmit message bits $m_1, m_2 ..., m_K$, $m_i \in \{+1, -1\}$, to the destination through orthogonal channels using BPSK modulation. The relay $R$, after overhearing $m_1, m_2 ...., m_K$ and decoding them correctly, generates a parity bit $p = a_1 m_1 \oplus a_2 m_2 \oplus ... \oplus a_k m_k$ where $\{a_i\}$ are network encoding coefficients. The LLR of $m_i$ at the destination is given by

$$L(m_i|\mathbf{h}, \mathbf{y}) = L(m_i|h_i, y_i) + L(p \oplus m_1 \oplus ... m_{i-1} \oplus m_{i+1} \oplus m_K|\mathbf{h}, \mathbf{y}) \tag{3.8}$$

where

$$L(p \oplus m_1 ... \oplus m_{i-1} \oplus m_{i+1} ... \oplus m_K|\mathbf{h}, \mathbf{y}) \approx \text{sgn}(L(p|h_r, y_r)) \prod_{1 \leq j \leq K, j \neq i} \text{sgn}(L(m_j|h_j, y_j)) \cdot \min_{1 \leq j \leq K, j \neq i} \{|L(m_j|h_j, y_j)|, |L(p|h_r, y_r)|\} \tag{3.9}$$

Hence, the magnitude of the total LLR is bounded by

$$|L(m_i|\mathbf{h}, \mathbf{y})| \leq |L(m_i|h_i, y_i)| \; + \min_{j \neq i}\{|L(m_j|h_j, y_j)|, |L(p|h_r, y_r)|\} \tag{3.10}$$

Since the reliability gain provided by the relay is determined by the second term in Eq.(3.10), the SPNC-I encoding rule for $K$ sources is

$$p = \begin{cases} m_1 \oplus m_i \oplus m_j & \text{if } g_i \geq \beta_1, g_j \geq \beta_1, \text{ for some } \; i, j \in, \{2, .., K\} \\ m_1 & \text{if } g_j < \beta_1, \;\; \text{for all j} \in \{2.....K\} \end{cases} \tag{3.11}$$

## 3.3 Soft Prioritized Network Coding II

The LLR for $m_2$ at D for given $\mathbf{h}$ and $\mathbf{y}$ is given by

$$L(m_2|\mathbf{h}, \mathbf{y}) = L(m_2|h_2, y_2) + L(p \oplus m_1|h_r, y_r, h_1, y_r) \tag{3.12}$$

It can be shown that the sign of $L(m_2|\mathbf{h}, \mathbf{y})$ is determined by $L(m_2|h_2, y_2)$ if

$$|L(m_2|h_2, y_2)| > |L(p \oplus m_1|h_r, y_r, h_1, y_r)| \tag{3.13}$$

This means that the MAP detection on $m_2$ does not depend on the additional information sent by the relay, i.e $L(p \oplus m_1|h_r, y_r, h_1, y_1)$, if Eq.(3.13) is satisfied. Hence, the relay may not combine $m_2$ without affecting its reliability at the destination when Eq.(3.13) is true. This observation motivates to consider the following encoding rule

$$p = \begin{cases} m_1 \oplus m_2 & \text{if } |L(m_2|h_2, y_2)| < |L(p \oplus m_1|h_r, y_r, h_1, y_r)| \\ m_1 & \text{if } |L(m_2|h_2, y_2)| \geq |L(p \oplus m_1|h_r, y_r, h_1, y_r)| \end{cases} \tag{3.14}$$

Since the LLR magnitude can be approximated by the SNR, Eq.(3.14) leads to following encoding rule

$$p = \begin{cases} m_1 \oplus m_2 & \text{if } g_2 < \min\{g_r, g_1\} \\ m_1 & \text{if } g_2 \geq \min\{g_r, g_1\} \end{cases} \tag{3.15}$$

where $g_i = d_i^{-\alpha}\ |h_i|^2$ and the approximations $|L(m_i|h_i, y_i)| \approx g_i\ E_i/N_0$ and

$$|L(p \oplus m_1|h_r, y_r, h_1, y_r)| \approx \min\{|L(p|h_r, y_r)|, |L(m_1|h_1, y_r)|\} \tag{3.16}$$

are applied. To provide a flexible QoS, the above coding rule can be modified as

$$p = \begin{cases} m_1 \oplus m_2 & \text{if } g_2 < \mu \cdot \min\{g_1, g_r\} \\ m_1 & \text{if } g_2 \geq \mu \cdot \min\{g_1, g_r\} \end{cases} \tag{3.17}$$

where $\mu$ is a constant which can be adjusted by the relay to provide appropriate QoS to the two users. This will be referred to as *Soft Prioritized Network Coding II* (SPNC-II).

## 3.4 Mathematical Analysis

In this section the probability of bit error for SPNC-I and SPNC-II for the case of two sources, one relay and one destination is derived. However, the analysis can be readily extended to multi-source scenario. Without loss of generality assume that $S_1$ is the prioritized source.

### 3.4.1 SPNC-I

The average bit error probability for source $S_i$, $i = 1,2$ is given by

$$P_{e,i} = P_{e,i|p=m_1 \oplus m_2} \cdot P_{p=m_1 \oplus m_2} + P_{e,i|p=m_1} \cdot P_{p=m_1} \tag{3.18}$$

where $P_{e,i|p=m_1 \oplus m_2}$ is the conditional bit error probability of source $S_i$ given $p = m_1 \oplus m_2$, $P_{e,i|p=m_1}$ is the conditional bit error probability of source $S_i$ given $p = m_1$, $P_{p=m_1 \oplus m_2}$ is the probability of transmission of $p = m_1 \oplus m_2$ and $P_{p=m_1}$ is the probability of transmission of $p = m_1$.

$$\begin{aligned} P_{p=m_1} &= Pr(|h_2|^2 \leq \beta_2) = 1 - e^{-\beta_2} \\ P_{p=m_1 \oplus m_2} &= Pr(|h_2|^2 \geq \beta_2) = e^{-\beta_2} \end{aligned} \tag{3.19}$$

where $\beta_2 = d_2^{\alpha}\, \beta_1$. Let $\mathbf{c} = (m_1, m_2, p)$ and $\hat{\mathbf{c}} = (\hat{m}_1, \hat{m}_2, \hat{p})$, be two distinct codewords ($\mathbf{c} \neq \hat{\mathbf{c}}$). Then, the pairwise error probability is given by (14)

$$\begin{aligned} P(\mathbf{c} \rightarrow \hat{\mathbf{c}}|\mathbf{h}) &= Q\left(\sqrt{\frac{\|\mathbf{h} \cdot (\mathbf{c} - \hat{\mathbf{c}})\|^2}{2N_0}}\right) \\ &= \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\|\mathbf{h} \cdot (\mathbf{c} - \hat{\mathbf{c}})\|^2}{4N_0 \sin^2\phi}\right) d\phi \end{aligned} \tag{3.20}$$

where $Q(x)$ denotes the Gaussian Q-function, $\|\mathbf{h} \cdot (\mathbf{c} - \hat{\mathbf{c}})\|^2 = |h_1|^2|m_1 - \hat{m}_1|^2 d_1^{-\alpha} E_s + |h_2|^2|m_2 - \hat{m}_2|^2 d_2^{-\alpha} E_s + |h_r|^2|p - \hat{p}|^2 d_r^{-\alpha} E_r$, and the second equation follows from Craig's formula (15)

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2sin^2\phi}} d\phi \tag{3.21}$$

Without loss of generality assume that $m_1 = 1$, $m_2 = 1$, $p = m_1 \oplus m_2 = 0$ was transmitted. Then, the conditional bit error probability for $S_i$ with the maximum likelihood decoding (ML) given $p = m_1 \oplus m_2$, i.e. $|h_2|^2 > \beta_2$ is bounded by

$$\begin{aligned} P_{e,i|p=m_1 \oplus m_2}(\mathbf{h}) \leq\; & Q\left(\sqrt{2|h_1|^2\gamma_1 + 2|h_2|^2\gamma_2}\,\Big|\,|h_2|^2 > \beta_2\right) \\ & + \frac{1}{2}Q\left(\sqrt{2|h_1|^2\gamma_1 + 2|h_r|^2\gamma_r}\,\Big|\,|h_2|^2 > \beta_2\right) \\ & + \frac{1}{2}Q\left(\sqrt{2|h_2|^2\gamma_2 + 2|h_r|^2\gamma_r}\,\Big|\,|h_2|^2 > \beta_2\right) \end{aligned} \tag{3.22}$$

where $\gamma_1 = d_1^{-\alpha} E_1/N_0$, $\gamma_2 = d_2^{-\alpha} E_2/N_0$ and $\gamma_r = d_r^{-\alpha} E_r/N_0$ are the receive SNRs at the destination from $S_1$, $S_2$ and $R$, respectively.

$$P_{e,i|p=m_1\oplus m_2} = \int_0^{\pi/2}\left[\frac{e^{-\beta_2\gamma_2\csc^2\phi}}{\pi\left(1+\gamma_1\csc^2\phi\right)\left(1+\gamma_2\csc^2\phi\right)} + \frac{e^{-\beta_2\gamma_2\csc^2\phi}}{2\pi\left(1+\gamma_r\csc^2\phi\right)\left(1+\gamma_2\csc^2\phi\right)}\right]d\phi$$
$$+\frac{\left(\sqrt{\gamma_1\gamma_r}+\sqrt{(1+\gamma_1)\gamma_r}+\sqrt{\gamma_1(1+\gamma_r)}\right)}{4\left(\sqrt{\gamma_1}+\sqrt{1+\gamma_1}\right)\sqrt{(1+\gamma_1)(1+\gamma_r)}\left(\sqrt{\gamma_r}+\sqrt{1+\gamma_r}\right)\left(\sqrt{(1+\gamma_1)\gamma_r}+\sqrt{\gamma_1(1+\gamma_r)}\right)} \tag{3.27}$$

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The probability distribution function (pdf) of random variable $X_i = |h_i|^2$, is given by

$$f_{X_i}(x_i) = e^{-x_i} \tag{3.23}$$

for $i = 1, 2, r$. The conditional distribution of $X_2 = |h_2|^2$ given that event $B = (X_2 > \beta_2)$ occurs is given by

$$f_{X_2}(x_2|B) = e^{-(x_2-\beta_2)} \tag{3.24}$$

Now averaging Eq.(3.22) over $f_{X_1}(x_1)$, $f_{X_r}(x_r)$ and $f_{X_2}(x_2|B)$ by integrating with respect to $x_1$, $x_2$ and $x_r$, and using Craig's formula we obtain

$$\begin{aligned} P_{e,i|p=m_1\oplus m_2} \leq & \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty}\left[e^{-x_1(\gamma_1\csc^2\phi+1)}dx_1\int_{\beta_2}^{\infty} e^{\beta_2}e^{-x_2(\gamma_2\csc^2\phi+1)}dx_2\right] \\ & + \frac{1}{2\pi}\int_0^{\pi/2} d\phi \int_0^{\infty}\left[e^{-x_r(\gamma_r\csc^2\phi+1)}dx_r\int_{\beta_2}^{\infty} e^{\beta_2}e^{-x_2(\gamma_2\csc^2\phi+1)}dx_2\right] \\ & + \frac{1}{2\pi}\int_0^{\pi/2} d\phi \int_0^{\infty}\left[e^{-x_1(\gamma_1\csc^2\phi+1)}dx_1\int_0^{\infty} e^{-x_r(\gamma_r\csc^2\phi+1)}dx_r\right] \end{aligned} \tag{3.25}$$

$$\begin{aligned} P_{e,i|p=m_1\oplus m_2} \leq & \frac{1}{\pi}\int_0^{\pi/2}\frac{e^{-\beta_2\gamma_2\csc^2\phi}}{\left(1+\gamma_1\csc^2\phi\right)\left(1+\gamma_2\csc^2\phi\right)}d\phi \\ & + \frac{1}{2\pi}\int_0^{\pi/2}\frac{e^{-\beta_2\gamma_2\csc^2\phi}}{\left(1+\gamma_r\csc^2\phi\right)\left(1+\gamma_2\csc^2\phi\right)}d\phi \\ & + \frac{1}{2\pi}\int_0^{\pi/2}\frac{1}{\left(1+\gamma_1\csc^2\phi\right)\left(1+\gamma_r\csc^2\phi\right)}d\phi \end{aligned} \tag{3.26}$$

The first and second terms in Eq.(3.26) cannot be obtained in closed form, however can be easily evaluated using numerical integration. The integration of the third term using Mathematica® software tool yields Eq.(3.27).

When $|h_2|^2 \leq \beta_2$, the relay does not include $m_2$ in the parity generation and thus $p = m_1$. The destination applies the maximal ratio combining (MRC) in decoding $m_1$.

$$P_{e,1|p=m_1} = \mathbb{E}_{h_1,h_r}\left[Q\left(\sqrt{2|h_1|^2\gamma_1 + 2|h_r|^2\gamma_r}\right)\right] \tag{3.28}$$

where $\mathbb{E}_{h_1,h_r}$ denotes expectation with respect to $h_1$ and $h_r$. The average bit error probability for $S_1$ can be derived using the Craig's formula

$$\begin{aligned} P_{e,1|p=m_1} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty}\left[e^{-x_1(\gamma_1 \csc^2\phi+1)}dx_1 \int_0^{\infty} e^{-x_r(\gamma_r \csc^2\phi+1)}dx_r\right] \\ &= \int_0^{\pi/2} \frac{1}{\pi(1+\gamma_1\csc^2\phi)(1+\gamma_r\csc^2\phi)}d\phi \\ &= \frac{\gamma_1-\gamma_r-\frac{(\gamma_1)^{3/2}}{\sqrt{1+\gamma_1}}+\frac{(\gamma_r)^{3/2}}{\sqrt{1+\gamma_r}}}{2(\gamma_1-\gamma_r)} \end{aligned} \tag{3.29}$$

When $|h_2|^2 \le \beta_2$, D receives the information about $m_2$ from $S_2$ only. Hence, the conditional bit error probability for $S_2$ given $h_2$ is given by

$$P_{e,2|p=m_1}(h_2) = Q\left(\sqrt{2|h_2|^2\gamma_2}\right) \tag{3.30}$$

The average bit error probability for $S_2$, conditioned on $|h_2|^2 \le \beta_2$ can be calculated using Craig's formula

$$\begin{aligned} P_{e,2|p=m_1} &= \int_0^{\beta_2} Q\left(\sqrt{2x_2\gamma_2}\right)\frac{e^{-x_2}}{P(|h_2|^2<\beta_2)}dx_2 \\ &= \frac{1}{\pi(1-e^{-\beta_2})}\int_0^{\pi/2} d\phi \int_0^{\beta_2} e^{-x_2\left(1+\gamma_2\csc^2\phi\right)}dx_2 \\ &= \frac{1}{\pi(1-e^{-\beta_2})}\int_0^{\pi/2}\frac{1-e^{-\beta_2\left(1+\gamma_2\csc^2\phi\right)}}{1+\gamma_2\csc^2\phi}d\phi \end{aligned} \tag{3.31}$$

Now using Eqs.(3.18), (3.19), (3.27) and (3.29), the average bit error probability of prioritized source $S_1$ can be found out. By using Eqs.(3.18), (3.19), (3.27) and (3.31), the average bit error probability of non-prioritized source $S_2$ can be found out.

### 3.4.2 SPNC-II

In this section, we will derive bit error probabilities for $S_1$ and $S_2$ when SPNC-II coding scheme is used at the relay. Let $x_1 = g_1$, $x_2 = g_2/\mu$, $x_r = g_r$. Also defining following events

- $\mathbf{Z}_1 = \{(x_1, x_2, x_r)|x_1 > x_2 > x_r\}$,
- $\mathbf{Z}_2 = \{(x_1, x_2, x_r)|x_2 > x_1 > x_r\}$,
- $\mathbf{Z}_3 = \{(x_1, x_2, x_r)|x_r > x_2 > x_1\}$,

- $\mathbf{Z}_4 = \{(x_1, x_2, x_r)|x_2 > x_r > x_1\}$,
- $\mathbf{Z}_5 = \{(x_1, x_2, x_r)|x_1 > x_r > x_2\}$,
- $\mathbf{Z}_6 = \{(x_1, x_2, x_r)|x_r > x_1 > x_2\}$

which denote the mutually exclusive events that cover the probability space spanned by random variables $x_1$, $x_2$ and $x_r$.

In the events of $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_4$, $x_2 \geq \min\{x_1, x_r\}$ occurs, hence the relay transmits $p = m_1$. In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, $x_2 < \min\{x_1, x_r\}$ occurs, hence the relay transmits $p = m_1 \oplus m_2$. The probability distribution of random variables $x_1$, $x_2$ and $x_r$ are given by $d_1^\alpha e^{-d_1^\alpha x_1}$, $\mu d_2^\alpha e^{-\mu d_2^\alpha x_2}$ and $d_r^\alpha e^{-d_r^\alpha x_r}$. The random variables are independent and hence the joint pdf is a product of marginals. The average bit error probability for source $S_i$, i = 1, 2 is given by

$$P_{e,i} = \sum_{j=1}^{6} P_{e,i,\mathbf{Z}_j} \tag{3.32}$$

where $P_{e,i,\mathbf{Z}_j}$ is the bit error probability for $S_i$ under the event $\mathbf{Z}_j$. In the events of $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, the relay transmits $p = m_1$ and the destination then uses MRC to detect $m_1$. Hence the conditional bit error probability for $m_1$ given $(\underline{\mathbf{x}}) = \{x_1, x_2, x_r\}$ is given by

$$P_{e,1,\mathbf{Z}_j}(\underline{\mathbf{x}}) = Q\left(\sqrt{2x_1 E_1/N_0 + 2x_r E_r/N_0}\right) \tag{3.33}$$

where j = 1, 2, 3, 4. Averaging Eq.(3.33) over $x_1$, $x_2$ and $x_r$ yields

$$P_{e,1,\mathbf{Z}_j} = \iiint\limits_{\mathbf{Z}_j} P_{e,1,\mathbf{Z}_j}(\underline{\mathbf{x}}) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{3.34}$$

where $f(\underline{\mathbf{x}}) = \mu d_1^\alpha d_2^\alpha d_r^\alpha e^{-(d_1^\alpha x_1 + \mu d_2^\alpha \cdot x_2 + d_r^\alpha x_r)}$. Using limit of integration for $\mathbf{Z}_1$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_1} &= \int_0^{\pi/2} \frac{d\phi}{\pi} \int_0^{\infty} d_1^\alpha e^{-x_1 d_1^\alpha(\gamma_1 \csc^2\phi + 1)} dx_1 \int_0^{x_1} \mu d_2^\alpha e^{-\mu d_2^\alpha x_2} dx_2 \int_0^{x_2} d_r^\alpha e^{-x_r d_r^\alpha(\gamma_r \csc^2\phi + 1)} dx_r \\ &= \frac{1}{\pi} \int_0^{\pi/2} \left[ \frac{\mu d_2^\alpha}{(1 + \gamma_1 \csc^2\phi)\left((1 + \gamma_1 \csc^2\phi)\, d_1^\alpha + \mu d_2^\alpha\right)} \times \right. \\ &\qquad \left. \frac{d_r^\alpha}{\left((1 + \gamma_1 \csc^2\phi)\, d_1^\alpha + \mu d_2^\alpha + (1 + \gamma_r \csc^2\phi)\, d_r^\alpha\right)} \right] d\phi \end{aligned} \tag{3.35}$$

Using limit of integration for $\mathbf{Z}_2$ and using Craig's formula we obtain

$$P_{e,1,\mathbf{Z}_2} = \int_0^{\pi/2} \frac{d\phi}{\pi} \int_0^{\infty} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2} dx_2 \int_0^{x_2} d_1^{\alpha} e^{-x_1 d_1^{\alpha}(\gamma_1 \csc^2\phi+1)} dx_1 \int_0^{x_1} d_r^{\alpha} e^{-x_r d_r^{\alpha}(\gamma_r \csc^2\phi+1)} dx_r$$
$$= \frac{1}{\pi}\int_0^{\pi/2} \frac{d_1^{\alpha} d_r^{\alpha}}{\left((1+\gamma_1\csc^2\phi)\,d_1^{\alpha}+\mu d_2^{\alpha}\right)\left((1+\gamma_1\csc^2\phi)\,d_1^{\alpha}+\mu d_2^{\alpha}+(1+\gamma_r\csc^2\phi)\,d_r^{\alpha}\right)} d\phi \tag{3.36}$$

Using limit of integration for $\mathbf{Z}_3$ and using Craig's formula we obtain

$$P_{e,1,\mathbf{Z}_3} = \int_0^{\pi/2} \frac{d\phi}{\pi} \int_0^{\infty} d_r^{\alpha} e^{-x_r d_r^{\alpha}(\gamma_r \csc^2\phi+1)} dx_r \int_0^{x_r} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2} dx_2 \int_0^{x_2} d_1^{\alpha} e^{-x_1 d_1^{\alpha}(\gamma_1 \csc^2\phi+1)} dx_1$$
$$= \frac{1}{\pi}\int_0^{\pi/2} \Bigg[\frac{\mu d_1^{\alpha}}{(1+\gamma_r\csc^2\phi)\left(\mu d_2^{\alpha}+(1+\gamma_r\csc^2\phi)\,d_r^{\alpha}\right)}\times$$
$$\frac{d_2^{\alpha}}{\left((1+\gamma_1\csc^2\phi)\,d_1^{\alpha}+\mu d_2^{\alpha}+(1+\gamma_r\csc^2\phi)\,d_r^{\alpha}\right)}\Bigg] d\phi \tag{3.37}$$

Using limit of integration for $\mathbf{Z}_4$ and using Craig's formula we obtain

$$P_{e,1,\mathbf{Z}_4} = \int_0^{\pi/2} \frac{d\phi}{\pi} \int_0^{\infty} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2} dx_2 \int_0^{x_2} d_r^{\alpha} e^{-x_r d_r^{\alpha}(\gamma_r \csc^2\phi+1)} dx_r \int_0^{x_r} d_1^{\alpha} e^{-x_1 d_1^{\alpha}(\gamma_1 \csc^2\phi+1)} dx_1$$
$$= \frac{1}{\pi}\int_0^{\pi/2} \frac{d_1^{\alpha} d_r^{\alpha}}{\left(\mu d_2^{\alpha}+(1+\gamma_r\csc^2\phi)\,d_r^{\alpha}\right)\left((1+\gamma_1\csc^2\phi)\,d_1^{\alpha}+\mu d_2^{\alpha}+(1+\gamma_r\csc^2\phi)\,d_r^{\alpha}\right)} d\phi \tag{3.38}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect $m_1$ and $m_2$. The union bound is used to evaluate a tight upper bound on bit error rate in those cases which can be expressed as

$$P_{e,1,\mathbf{Z}_5}(\underline{\mathbf{x}}) \leq Q\left(\sqrt{2x_1E_1/N_0+2\mu x_2E_2/N_0}\right)+\frac{1}{2}Q\left(\sqrt{2x_1E_1/N_0+2x_rE_r/N_0}\right)$$
$$+\frac{1}{2}Q\left(\sqrt{2\mu x_2E_2/N_0+2x_rE_r/N_0}\right) \tag{3.39}$$

Averaging over joint distribution of $x_1$, $x_2$ and $x_r$ for event $\mathbf{Z}_5$ and using Craig's formula we obtain

$$\begin{aligned}
P_{e,1,\mathbf{Z}_5} \leq & \int_0^{\pi/2} \frac{d\phi}{\pi} \int_0^{\infty} d_1^{\alpha} e^{-d_1^{\alpha} x_1 (\gamma_1 \csc^2\phi + 1)} dx_1 \int_0^{x_1} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \int_0^{x_r} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2 (\gamma_2 \csc^2\phi + 1)} dx_2 \\
& + \int_0^{\pi/2} \frac{d\phi}{2\pi} \int_0^{\infty} d_1^{\alpha} e^{-d_1^{\alpha} x_1 (\gamma_1 \csc^2\phi + 1)} dx_1 \int_0^{x_1} d_r^{\alpha} e^{-d_r^{\alpha} x_r (\gamma_r \csc^2\phi + 1)} dx_r \int_0^{x_r} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2} dx_2 \\
& + \int_0^{\pi/2} \frac{d\phi}{2\pi} \int_0^{\infty} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \int_0^{x_1} e^{-d_r^{\alpha} x_r (\gamma_r \csc^2\phi + 1)} dx_r \int_0^{x_r} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2 (\gamma_2 \csc^2\phi + 1)} dx_2 \\
\leq & \frac{1}{\pi} \int_0^{\pi/2} \left[ \frac{\mu d_2^{\alpha}}{(1+\gamma_1 \csc^2\phi)\left((1+\gamma_1\csc^2\phi) d_1^{\alpha} + d_r^{\alpha}\right)} \times \right. \\
& \left. \frac{d_r^{\alpha}}{\left((1+\gamma_1\csc^2\phi) d_1^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2\csc^2\phi) + d_r^{\alpha}\right)} \right] d\phi \\
& + \frac{1}{2\pi} \int_0^{\pi/2} \left[ \frac{\mu d_2^{\alpha}}{(1+\gamma_1\csc^2\phi)\left(d_1^{\alpha}(1+\gamma_1\csc^2\phi) + d_r^{\alpha}(1+\gamma_r\csc^2\phi)\right)} \times \right. \\
& \left. \frac{d_r^{\alpha}}{\left(d_1^{\alpha}(1+\gamma_1\csc^2\phi) + \mu d_2^{\alpha} + d_r^{\alpha}(1+\gamma_r\csc^2\phi)\right)} \right] d\phi \\
& + \frac{1}{2\pi} \int_0^{\pi/2} \left[ \frac{\mu d_2^{\alpha} d_r^{\alpha}}{\left(d_1^{\alpha} + d_r^{\alpha}(1+\gamma_r\csc^2\phi)\right)\left(d_1^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2\csc^2\phi) + d_r^{\alpha}(1+\gamma_r\csc^2\phi)\right)} \right] d\phi
\end{aligned} \tag{3.40}$$

Similarly, since the relay transmits $p = m_1 \oplus m_2$, for the event $\mathbf{Z}_6$, union bound is used to derive $P_{e,1,\mathbf{Z}_6}$.

$$\begin{aligned}
P_{e,1,\mathbf{Z}_6} \leq & \int_0^{\pi/2} \frac{d\phi}{\pi} \int_0^{\infty} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \int_0^{x_r} d_1^{\alpha} e^{-d_1^{\alpha} x_1 (\gamma_1 \csc^2\phi + 1)} dx_1 \int_0^{x_1} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2 (\gamma_2 \csc^2\phi + 1)} dx_2 \\
& + \int_0^{\pi/2} \frac{d\phi}{2\pi} \int_0^{\infty} d_r^{\alpha} e^{-d_r^{\alpha} x_r (\gamma_r \csc^2\phi + 1)} dx_r \int_0^{x_r} d_1^{\alpha} e^{-d_1^{\alpha} x_1 (\gamma_1 \csc^2\phi + 1)} dx_1 \int_0^{x_1} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2} dx_2 \\
& + \int_0^{\pi/2} \frac{d\phi}{2\pi} \int_0^{\infty} d_r^{\alpha} e^{-d_r^{\alpha} x_r (\gamma_r \csc^2\phi + 1)} dx_r \int_0^{x_r} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \int_0^{x_1} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2 (\gamma_2 \csc^2\phi + 1)} dx_2 \\
\leq & \frac{1}{\pi} \int_0^{\pi/2} \frac{\mu d_1^{\alpha} d_2^{\alpha}}{\left((1+\gamma_1\csc^2\phi) d_1^{\alpha} + d_r^{\alpha}\right)\left((1+\gamma_1\csc^2\phi) d_1^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2\csc^2\phi) + d_r^{\alpha}\right)} d\phi \\
& + \frac{1}{2\pi} \int_0^{\pi/2} \left[ \frac{\mu d_1^{\alpha}}{(1+\gamma_r\csc^2\phi)\left(d_1^{\alpha}(1+\gamma_1\csc^2\phi) + d_r^{\alpha}(1+\gamma_r\csc^2\phi)\right)} \times \right. \\
& \left. \frac{d_2^{\alpha}}{\left(d_1^{\alpha}(1+\gamma_1\csc^2\phi) + \mu d_2^{\alpha} + d_r^{\alpha}(1+\gamma_r\csc^2\phi)\right)} \right] d\phi \\
& + \frac{1}{2\pi} \int_0^{\pi/2} \left[ \frac{\mu d_1^{\alpha}}{(1+\gamma_r\csc^2\phi)\left(d_1^{\alpha} + (1+\gamma_r\csc^2\phi) d_r^{\alpha}\right)} \times \right. \\
& \left. \frac{d_2^{\alpha}}{\left(d_1^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2\csc^2\phi) + d_r^{\alpha}(1+\gamma_r\csc^2\phi)\right)} \right] d\phi
\end{aligned} \tag{3.41}$$

Using Equations (3.32), (3.35), (3.36), (3.37), (3.38), (3.40), (3.41), bound on $P_{e,1}$ can be found. This bound is very accurate as shown in simulation results.

Now $P_{e,2,\mathbf{Z}_1}$, $P_{e,2,\mathbf{Z}_2}$, $P_{e,2,\mathbf{Z}_3}$, $P_{e,2,\mathbf{Z}_4}$ will be derived. For events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, the destination decodes $m_2$ using the signal received directly from $S_2$. Hence the bit error probability for $m_2$ for events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$ is given by

$$P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) = Q\left(\sqrt{2\mu x_2 E_2/N_0}\right) \tag{3.42}$$

where j = 1, 2, 3, 4. Averaging Eq.(3.42) over $x_1$, $x_2$ and $x_r$ yields

$$P_{e,2,\mathbf{Z}_j} = \iiint_{\mathbf{Z}_j} P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{3.43}$$

Using limit of integration for $\mathbf{Z}_j$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,2,\mathbf{Z}_1} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \int_0^{x_1} \mu d_2^{\alpha} e^{-\mu x_2 d_2^{\alpha}(\gamma_2 \csc^2\phi + 1)} dx_2 \int_0^{x_2} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \\ &= \frac{1}{\pi}\int_0^{\pi/2} \frac{\mu d_2^{\alpha} d_r^{\alpha}}{\left(d_1^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)} d\phi \end{aligned} \tag{3.44}$$

Using limit of integration for $\mathbf{Z}_2$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,2,\mathbf{Z}_2} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2(\gamma_2 \csc^2\phi + 1)} dx_2 \int_0^{x_2} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \int_0^{x_1} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \\ &= \frac{1}{\pi}\int_0^{\pi/2} \frac{d_1^{\alpha} d_r^{\alpha}}{\left(1+\gamma_2\csc^2\phi\right)\left(d_1^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)} d\phi \end{aligned} \tag{3.45}$$

Using limit of integration for $\mathbf{Z}_3$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,2,\mathbf{Z}_3} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \int_0^{x_r} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2(\gamma_2 \csc^2\phi + 1)} dx_2 \int_0^{x_2} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \\ &= \frac{1}{\pi}\int_0^{\pi/2} \frac{\mu d_1^{\alpha} d_2^{\alpha}}{\pi\left(d_r^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)} d\phi \end{aligned} \tag{3.46}$$

Using limit of integration for $\mathbf{Z}_4$ and using Craig's formula we obtain

$$\begin{aligned} P_{e,2,\mathbf{Z}_4} &= \frac{1}{\pi}\int_0^{\pi/2} d\phi \int_0^{\infty} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2(\gamma_2 \csc^2\phi + 1)} dx_2 \int_0^{x_2} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \int_0^{x_r} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \\ &= \frac{1}{\pi}\int_0^{\pi/2} \frac{d_1^{\alpha} d_r^{\alpha}}{\left(1+\gamma_2\csc^2\phi\right)\left(d_r^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}\left(1+\gamma_2\csc^2\phi\right)\right)} d\phi \end{aligned} \tag{3.47}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect $m_1$ and $m_2$, hence we have $P_{e,2,\mathbf{Z}_5} = P_{e,1,\mathbf{Z}_5}$ and $P_{e,2,\mathbf{Z}_6} = P_{e,1,\mathbf{Z}_6}$. Using Equations (3.40), (3.41), (3.32), (3.44), (3.45), (3.46), (3.47), bound on $P_{e,2}$ can be found. This bound is also very accurate as shown in simulation results.

### 3.4.3 Q function upper bound on Bit Error Probability

The bit error probability for SPNC-II scheme derived in previous section matches quite well with simulation results. However, it doesn't gives a very good idea about the diversity order and the asymptotic behavior of coding scheme because of non-closed form expressions. This is primarily because of using Craig's formula. In this section we derive bit error probability using bound on Q-function and as shown in simulation results, the bound is fairly tight.

We have the following bound on Q function:

$$Q(x) \leq \frac{1}{2} e^{-\frac{x^2}{2}} \tag{3.48}$$

Using this bound we will rederive the bit error probability expressions for $S_1$ and $S_2$. From Eq.(3.33) we have

$$P_{e,1,\mathbf{Z}_j}(x_1, x_2, x_r) = Q\left(\sqrt{2x_1 E_1/N_0 + 2x_r E_r/N_0}\right) \tag{3.49}$$

where j = 1, 2, 3, 4. Averaging Eq.(3.49) over $x_1$, $x_2$ and $x_r$ yields

$$P_{e,1,\mathbf{Z}_j} = \iiint\limits_{\mathbf{Z}_j} Q\left(\sqrt{2x_1 E_1/N_0 + 2x_r E_r/N_0}\right) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{3.50}$$

where $f(\underline{\mathbf{x}}) = \mu d_1^\alpha d_2^\alpha d_r^\alpha e^{-(d_1^\alpha x_1 + \mu d_2^\alpha \cdot x_2 + d_r^\alpha x_r)}$. Using limit of integration for $\mathbf{Z}_j$ and Q function bound, Eq.(3.48), we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_1} &\leq \frac{1}{2} \int_0^\infty d_1^\alpha e^{-x_1 d_1^\alpha(\gamma_1+1)} dx_1 \int_0^{x_1} \mu d_2^\alpha e^{-\mu d_2^\alpha x_2} dx_2 \int_0^{x_2} d_r^\alpha e^{-x_r d_r^\alpha(\gamma_r+1)} dx_r \\ &\leq \frac{1}{2} \frac{\mu d_2^\alpha d_r^\alpha}{(1+\gamma_1)\left((1+\gamma_1) d_1^\alpha + \mu d_2^\alpha\right)\left((1+\gamma_1) d_1^\alpha + \mu d_2^\alpha + (1+\gamma_r) d_r^\alpha\right)} \end{aligned} \tag{3.51}$$

$$\begin{aligned} P_{e,1,\mathbf{Z}_2} &\leq \frac{1}{2} \int_0^\infty \mu d_2^\alpha e^{-\mu d_2^\alpha x_2} dx_2 \int_0^{x_2} d_1^\alpha e^{-x_1 d_1^\alpha(\gamma_1+1)} dx_1 \int_0^{x_1} d_r^\alpha e^{-x_r d_r^\alpha(\gamma_r+1)} dx_r \\ &\leq \frac{1}{2} \frac{d_1^\alpha d_r^\alpha}{\left((1+\gamma_1) d_1^\alpha + \mu d_2^\alpha\right)\left((1+\gamma_1) d_1^\alpha + \mu d_2^\alpha + (1+\gamma_r) d_r^\alpha\right)} \end{aligned} \tag{3.52}$$

$$\begin{aligned} P_{e,1,\mathbf{Z}_3} &\leq \frac{1}{2} \int_0^\infty d_r^\alpha e^{-x_r d_r^\alpha(\gamma_r+1)} dx_r \int_0^{x_r} \mu d_2^\alpha e^{-\mu d_2^\alpha x_2} dx_2 \int_0^{x_2} d_1^\alpha e^{-x_1 d_1^\alpha(\gamma_1+1)} dx_1 \\ &\leq \frac{1}{2} \frac{\mu d_1^\alpha d_2^\alpha}{(1+\gamma_r)\left(\mu d_2^\alpha + (1+\gamma_r) d_r^\alpha\right)\left((1+\gamma_1) d_1^\alpha + \mu d_2^\alpha + (1+\gamma_r) d_r^\alpha\right)} \end{aligned} \tag{3.53}$$

$$\begin{aligned} P_{e,1,\mathbf{Z}_4} &\leq \frac{1}{2} \int_0^\infty \mu d_2^\alpha e^{-\mu d_2^\alpha x_2} dx_2 \int_0^{x_2} d_r^\alpha e^{-x_r d_r^\alpha(\gamma_r+1)} dx_r \int_0^{x_r} d_1^\alpha e^{-x_1 d_1^\alpha(\gamma_1+1)} dx_1 \\ &\leq \frac{1}{2} \frac{d_1^\alpha d_r^\alpha}{\left(\mu d_2^\alpha + (1+\gamma_r) d_r^\alpha\right)\left((1+\gamma_1) d_1^\alpha + \mu d_2^\alpha + (1+\gamma_r) d_r^\alpha\right)} \end{aligned} \tag{3.54}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect $m_1$ and $m_2$. The union bound is used to evaluate a tight upper bound on bit error rate in those cases which can be expressed as

$$\begin{aligned} P_{e,1,\mathbf{Z}_5}(\underline{\mathbf{x}}) \leq & Q\left(\sqrt{2x_1E_1/N_0 + 2\mu x_2E_2/N_0}\right) + \frac{1}{2}Q\left(\sqrt{2x_1E_1/N_0 + 2x_rE_r/N_0}\right) \\ & + \frac{1}{2}Q\left(\sqrt{2\mu x_2E_2/N_0 + 2x_rE_r/N_0}\right) \end{aligned} \tag{3.55}$$

Averaging over joint distribution of $x_1$, $x_2$ and $x_r$ for event $\mathbf{Z}_5$ and using Q function bound we obtain

$$\begin{aligned} P_{e,1,\mathbf{Z}_5} \leq & \frac{1}{2}\int_0^{\infty} d_1^{\alpha}e^{-d_1^{\alpha}x_1(\gamma_1+1)}dx_1 \int_0^{x_1} d_r^{\alpha}e^{-d_r^{\alpha}x_r}dx_r \int_0^{x_r} \mu d_2^{\alpha}e^{-\mu d_2^{\alpha}x_2(\gamma_2+1)}dx_2 \\ & + \frac{1}{4}\int_0^{\infty} d_1^{\alpha}e^{-d_1^{\alpha}x_1(\gamma_1+1)}dx_1 \int_0^{x_1} d_r^{\alpha}e^{-d_r^{\alpha}x_r(\gamma_r+1)}dx_r \int_0^{x_r} \mu d_2^{\alpha}e^{-\mu d_2^{\alpha}x_2}dx_2 \\ & + \frac{1}{4}\int_0^{\infty} d_1^{\alpha}e^{-d_1^{\alpha}x_1}dx_1 \int_0^{x_1} e^{-d_r^{\alpha}x_r(\gamma_r+1)}dx_r \int_0^{x_r} \mu d_2^{\alpha}e^{-\mu d_2^{\alpha}x_2(\gamma_2+1)}dx_2 \\ \leq & \frac{1}{2}\left[\frac{\mu d_2^{\alpha}d_r^{\alpha}}{(1+\gamma_1)\left((1+\gamma_1)d_1^{\alpha}+d_r^{\alpha}\right)\left((1+\gamma_1)d_1^{\alpha}+\mu d_2^{\alpha}(1+\gamma_2)+d_r^{\alpha}\right)}\right] \\ & + \frac{1}{4}\left[\frac{\mu d_2^{\alpha}d_r^{\alpha}}{(1+\gamma_1)\left(d_1^{\alpha}(1+\gamma_1)+d_r^{\alpha}(1+\gamma_r)\right)\left(d_1^{\alpha}(1+\gamma_1)+\mu d_2^{\alpha}+d_r^{\alpha}(1+\gamma_r)\right)}\right] \\ & + \frac{1}{4}\left[\frac{\mu d_2^{\alpha}d_r^{\alpha}}{\left(d_1^{\alpha}+d_r^{\alpha}(1+\gamma_r)\right)\left(d_1^{\alpha}+\mu d_2^{\alpha}(1+\gamma_2)+d_r^{\alpha}(1+\gamma_r)\right)}\right] \end{aligned} \tag{3.56}$$

Similarly, since the relay transmits $p = m_1 \oplus m_2$, for the event $\mathbf{Z}_6$, union bound is used to derive $P_{e,1,\mathbf{Z}_6}$.

$$\begin{aligned} P_{e,1,\mathbf{Z}_6} \leq & \frac{1}{2}\int_0^{\infty} d_r^{\alpha}e^{-d_r^{\alpha}x_r}dx_r \int_0^{x_r} d_1^{\alpha}e^{-d_1^{\alpha}x_1(\gamma_1+1)}dx_1 \int_0^{x_1} \mu d_2^{\alpha}e^{-\mu d_2^{\alpha}x_2(\gamma_2+1)}dx_2 \\ & + \frac{1}{4}\int_0^{\infty} d_r^{\alpha}e^{-d_r^{\alpha}x_r(\gamma_r+1)}dx_r \int_0^{x_r} d_1^{\alpha}e^{-d_1^{\alpha}x_1(\gamma_1+1)}dx_1 \int_0^{x_1} \mu d_2^{\alpha}e^{-\mu d_2^{\alpha}x_2}dx_2 \\ & + \frac{1}{4}\int_0^{\infty} d_r^{\alpha}e^{-d_r^{\alpha}x_r(\gamma_r+1)}dx_r \int_0^{x_r} d_1^{\alpha}e^{-d_1^{\alpha}x_1}dx_1 \int_0^{x_1} \mu d_2^{\alpha}e^{-\mu d_2^{\alpha}x_2(\gamma_2+1)}dx_2 \\ \leq & \frac{1}{2}\left[\frac{\mu d_1^{\alpha}d_2^{\alpha}}{\left((1+\gamma_1)d_1^{\alpha}+d_r^{\alpha}\right)\left((1+\gamma_1)d_1^{\alpha}+\mu d_2^{\alpha}(1+\gamma_2)+d_r^{\alpha}\right)}\right] \\ & + \frac{1}{4}\left[\frac{\mu d_1^{\alpha}d_2^{\alpha}}{(1+\gamma_r)\left(d_1^{\alpha}(1+\gamma_1)+d_r^{\alpha}(1+\gamma_r)\right)\left(d_1^{\alpha}(1+\gamma_1)+\mu d_2^{\alpha}+d_r^{\alpha}(1+\gamma_r)\right)}\right] \\ & + \frac{1}{4}\left[\frac{\mu d_1^{\alpha}d_2^{\alpha}}{(1+\gamma_r)\left(d_1^{\alpha}+(1+\gamma_r)d_r^{\alpha}\right)\left(d_1^{\alpha}+\mu d_2^{\alpha}(1+\gamma_2)+d_r^{\alpha}(1+\gamma_r)\right)}\right] \end{aligned} \tag{3.57}$$

Now $P_{e,2,\mathbf{Z}_1}$, $P_{e,2,\mathbf{Z}_2}$, $P_{e,2,\mathbf{Z}_3}$, $P_{e,2,\mathbf{Z}_4}$ will be derived. For events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, the destination decodes $m_2$ using the signal received directly from $S_2$. Hence the bit error probability for $m_2$ for events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$ is given by

$$P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) = Q\left(\sqrt{2\mu x_2E_2/N_0}\right) \tag{3.58}$$

where j = 1, 2, 3, 4. Averaging Eq.(3.58) over $x_1$, $x_2$ and $x_r$ yields

$$P_{e,2,\mathbf{Z}_j} = \iiint\limits_{\mathbf{Z}_j} P_{e,2,\mathbf{Z}_j}(\underline{\mathbf{x}}) f(\underline{\mathbf{x}}) d\underline{\mathbf{x}} \tag{3.59}$$

Using limit of integration for $\mathbf{Z}_j$ and using Q function bound we obtain

$$\begin{aligned} P_{e,2,\mathbf{Z}_1} &\leq \frac{1}{2}\int_0^{\infty} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \int_0^{x_1} \mu d_2^{\alpha} e^{-\mu x_2 d_2^{\alpha}(\gamma_2+1)} dx_2 \int_0^{x_2} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \\ &\leq \frac{1}{2} \frac{\mu d_2^{\alpha} d_r^{\alpha}}{\left(d_1^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)} \end{aligned} \tag{3.60}$$

$$\begin{aligned} P_{e,2,\mathbf{Z}_2} &\leq \frac{1}{2}\int_0^{\infty} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2(\gamma_2+1)} dx_2 \int_0^{x_2} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \int_0^{x_1} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \\ &\leq \frac{1}{2} \frac{d_1^{\alpha} d_r^{\alpha}}{(1+\gamma_2)\left(d_1^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)} \end{aligned} \tag{3.61}$$

$$\begin{aligned} P_{e,2,\mathbf{Z}_3} &\leq \frac{1}{2}\int_0^{\infty} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \int_0^{x_r} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2(\gamma_2+1)} dx_2 \int_0^{x_2} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \\ &\leq \frac{1}{2} \frac{\mu d_1^{\alpha} d_2^{\alpha}}{\left(d_r^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)} \end{aligned} \tag{3.62}$$

$$\begin{aligned} P_{e,2,\mathbf{Z}_4} &\leq \frac{1}{2}\int_0^{\infty} \mu d_2^{\alpha} e^{-\mu d_2^{\alpha} x_2(\gamma_2+1)} dx_2 \int_0^{x_2} d_r^{\alpha} e^{-d_r^{\alpha} x_r} dx_r \int_0^{x_r} d_1^{\alpha} e^{-d_1^{\alpha} x_1} dx_1 \\ &\leq \frac{1}{2} \frac{d_1^{\alpha} d_r^{\alpha}}{(1+\gamma_2)\left(d_r^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)\left(d_1^{\alpha} + d_r^{\alpha} + \mu d_2^{\alpha}(1+\gamma_2)\right)} \end{aligned} \tag{3.63}$$

In the events of $\mathbf{Z}_5$ and $\mathbf{Z}_6$, the relay transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect $m_1$ and $m_2$, hence we have $P_{e,2,\mathbf{Z}_5} = P_{e,1,\mathbf{Z}_5}$ and $P_{e,2,\mathbf{Z}_6} = P_{e,1,\mathbf{Z}_6}$. Using Equations (3.56), (3.57), (3.32), (3.60), (3.61), (3.62), (3.63), bound on $P_{e,2}$ can be found. This bound is also very tight within a dB as shown in simulation results.

### 3.4.4 Asymptotic Analysis

In this section, we will derive asymptotic bounds for the bit error probabilities. Assume $\gamma_1 = \gamma_2 = \gamma$, $d_1 = d_2 = d$ and $(d_r/d)^{\alpha} = \delta$. Therefore $\gamma_r = \gamma/\delta$.

At high SNR ($\gamma \to \infty$), for $0 < \mu < \infty$, collecting dominant terms for $P_{e,1}$ we obtain following bound on $P_{e,1}$:

$$\begin{aligned} P_{e,1} \leq & \frac{1}{2} \frac{d_1^\alpha d_r^\alpha}{\left(\left(1+\gamma_1\right) d_1^\alpha + \mu d_2^\alpha\right)\left(\left(1+\gamma_1\right) d_1^\alpha + \mu d_2^\alpha + \left(1+\gamma_r\right) d_r^\alpha\right)} \\ & + \frac{1}{2} \frac{d_1^\alpha d_r^\alpha}{\left(\mu d_2^\alpha + \left(1+\gamma_r\right) d_r^\alpha\right)\left(\left(1+\gamma_1\right) d_1^\alpha + \mu d_2^\alpha + \left(1+\gamma_r\right) d_r^\alpha\right)} \\ & + \frac{1}{4} \frac{\mu d_2^\alpha d_r^\alpha}{\left(d_1^\alpha + d_r^\alpha\left(1+\gamma_r\right)\right)\left(d_1^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right) + d_r^\alpha\left(1+\gamma_r\right)\right)} \\ & + \frac{1}{2} \frac{\mu d_1^\alpha d_2^\alpha}{\left(\left(1+\gamma_1\right) d_1^\alpha + d_r^\alpha\right)\left(\left(1+\gamma_1\right) d_1^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right) + d_r^\alpha\right)} \end{aligned} \tag{3.64}$$

Eq. (3.64) can be expressed as:

$$\begin{aligned} P_{e,1} \leq & \frac{1}{2} \frac{\delta}{(\gamma+\mu+1)(2\gamma+\mu+\delta+1)} + \frac{1}{2} \frac{\delta}{(\gamma+\mu+\delta)(2\gamma+\mu+\delta+1)} \\ & + \frac{1}{4} \frac{\mu\delta}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} + \frac{1}{2} \frac{\mu}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} \end{aligned} \tag{3.65}$$

Clearly for high SNR, the diversity order for $S_1$ is 2. When $\mu > 2$, the bit error rate of $S_1$ is very close to the bit error rate of traditional network coding ($\mu = \infty$). For $0 < \mu < 2$, and high SNR ($\gamma \to \infty$), the asymptotic bound for bit error probability of $S_1$ is given by:

$$\begin{aligned} P_{e,1} & \leq \frac{\delta}{2\gamma^2} + \frac{\mu\delta}{4\gamma^2(1+\mu)} + \frac{\mu}{2\gamma^2(1+\mu)} \\ & = \left(\sqrt{\frac{4(1+\mu)}{2\delta+3\mu\delta+2\mu}} \times \gamma\right)^{-2} \end{aligned} \tag{3.66}$$

Now collecting dominant terms of $P_{e,2}$ for high SNR ($\gamma \to \infty$) and $0 < \mu < \infty$, following is the bound on $P_{e,2}$:

$$\begin{aligned} P_{e,2} \leq & \frac{1}{2} \frac{\mu d_2^\alpha d_r^\alpha}{\left(d_1^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right)\right)\left(d_1^\alpha + d_r^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right)\right)} \\ & + \frac{1}{2} \frac{\mu d_1^\alpha d_2^\alpha}{\left(d_r^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right)\right)\left(d_1^\alpha + d_r^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right)\right)} \\ & + \frac{1}{4} \frac{\mu d_2^\alpha d_r^\alpha}{\left(d_1^\alpha + d_r^\alpha\left(1+\gamma_r\right)\right)\left(d_1^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right) + d_r^\alpha\left(1+\gamma_r\right)\right)} \\ & + \frac{1}{2} \frac{\mu d_1^\alpha d_2^\alpha}{\left(\left(1+\gamma_1\right) d_1^\alpha + d_r^\alpha\right)\left(\left(1+\gamma_1\right) d_1^\alpha + \mu d_2^\alpha\left(1+\gamma_2\right) + d_r^\alpha\right)} \end{aligned} \tag{3.67}$$

Eq.(3.67) can be expressed as:

$$\begin{aligned} P_{e,2} \leq & \frac{1}{2} \frac{\mu\delta}{(1+\mu\gamma)(1+\delta+\mu\gamma)} + \frac{1}{2} \frac{\mu}{(\delta+\mu\gamma)(1+\delta+\mu\gamma)} \\ & + \frac{1}{4} \frac{\mu\delta}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} + \frac{1}{2} \frac{\mu}{(\gamma+\delta+1)(\gamma(1+\mu)+\mu+\delta+1)} \end{aligned} \tag{3.68}$$

Clearly for high SNR, the diversity order for $S_2$ is 2. When $\mu > 2$, the bit error rate of $S_2$ is very close to the bit error rate of traditional network coding ($\mu = \infty$). For $0 < \mu < 2$, and high SNR ($\gamma \to \infty$), the asymptotic bound for bit error probability of $S_2$ is given by:

$$\begin{aligned} P_{e,2} &\leq \frac{\delta}{2\mu\gamma^2} + \frac{1}{2\mu\gamma^2} + \frac{\mu\delta}{4\gamma^2(1+\mu)} + \frac{\mu}{2\gamma^2(1+\mu)} \\ &= \left(\sqrt{\frac{4\mu(1+\mu)}{2\mu^2+\mu^2\delta+2\mu\delta+2\mu+2\delta+2}} \times \gamma\right)^{-2} \end{aligned} \tag{3.69}$$

We will now derive approximate bit error probability for traditional network coding. When the relay performs traditional network coding, then it always transmits $p = m_1 \oplus m_2$. The destination then uses MAP decoding to detect $m_1$ and $m_2$. In this case the bit error probabilities for $S_1$ and $S_2$ are equal. The union bound is used to evaluate a tight upper bound on bit error rate which can be expressed as:

$$\begin{aligned} P_{NC}(\underline{\mathbf{x}}) \leq\ & Q\left(\sqrt{2x_1E_1/N_0 + 2\mu x_2E_2/N_0}\right) + \frac{1}{2}Q\left(\sqrt{2x_1E_1/N_0 + 2x_rE_r/N_0}\right) \\ &+ \frac{1}{2}Q\left(\sqrt{2\mu x_2E_2/N_0 + 2x_rE_r/N_0}\right) \end{aligned} \tag{3.70}$$

Using Q function approximation, Eq.(3.48), and averaging over joint distribution of $x_1$, $x_2$ and $x_r$ we obtain:

$$P_{NC} \leq \frac{1}{2\gamma_1\gamma_2} + \frac{1}{4\gamma_1\gamma_r} + \frac{1}{4\gamma_2\gamma_r} \tag{3.71}$$

For the case when $\gamma_1 = \gamma_2 = \gamma$, $d_1 = d_2 = d$ and $(d_r/d)^\alpha = \delta$, the bit error probability for network coding can be expressed as:

$$P_{NC} \leq \left(\sqrt{\frac{2}{\delta+1}} \times \gamma\right)^{-2} \tag{3.72}$$

For $\mu = 0.1$, $d_r/d = 0.5$ i.e $\delta = 0.0625$, we have

$$\begin{aligned} P_{e,1} &\leq (3.5777 \times \gamma)^{-2} \\ P_{e,2} &\leq (0.4320 \times \gamma)^{-2} \\ P_{NC} &\leq (1.3720 \times \gamma)^{-2} \end{aligned} \tag{3.73}$$

Hence the coding gain of $S_1$ over $S_2 \approx 10\times \log_{10}(3.5777/0.4320) = 9.18$ dB. The coding gain of $S_1$ for SPNC-II scheme compared to traditional network coding is $\approx 10\times \log_{10}(3.5777/1.3720)$ $= 4.16$ dB whereas the loss for $S_2$ is $\approx 10\times \log_{10}(1.3720/0.4320) = 5.02$ dB.

### 3.4.5 Effect of Channel Estimation Error

In practice, the channel estimation may not be perfect. The estimated channel fading gains can be modeled as (16; 18)

$$\hat{h}_i = h_i + e_i, \quad \text{i} = 1,\, 2,\, \text{r} \tag{3.74}$$

where $e_i$ is distributed as $\mathcal{CN}(0, \sigma_{e,i}^2)$ and represents the channel estimation error. For pilot symbol aided minimum mean squared error (MMSE) channel estimation, the error variance is given by (17)

$$\sigma_{e,i}^2 = \frac{1}{1 + \frac{\pi}{L\omega_{ND}} \cdot \frac{E_{ps,i}}{N_0}} \tag{3.75}$$

where L is the rate of insertion of pilot symbols, $E_{ps,i}$ is the average received energy per pilot symbol from the $i^{th}$ source at the destination and $\omega_{ND}$ is the normalized Doppler frequency, normalized with respect to sampling frequency. Then, the effective receive SNR is given by

$$\gamma_{e,i} = \frac{\gamma_i}{1 + \gamma_i \sigma_{e,i}^2} \tag{3.76}$$

which replaces $\gamma_i$ in Equations (3.35), (3.36), (3.37), (3.38), (3.40), (3.41) for $S_1$ when the channel estimation is not perfect and in Equations (3.40), (3.41), (3.44), (3.45), (3.46), (3.47) for $S_2$.

### 3.4.6 Outage Probability for SPNC - II

The outage probability for SPNC II scheme will be derived in this section. We first consider a point to point communication system as the base line communication system with spectral efficiency of R bits per channel use. The received signal at the destination is given by:

$$y = hx + n \tag{3.77}$$

The instantaneous signal to noise ratio (SNR) of the channel is given by $\Gamma = |h|^2 \, d^{-\alpha} E_s/N_0$, $|h|^2$ is exponentially distributed. When the instantaneous SNR is less than certain threshold, the source is said to be in *outage*. The outage probability is:

$$\begin{aligned} P_{out} &= P\left(\Gamma < 2^R - 1\right) \\ &= P\left(d^{-\alpha}|h|^2 < \frac{2^R - 1}{\frac{E_s}{N_0}}\right) \\ &= 1 - e^{\left(-\frac{2^R-1}{\gamma}\right)} \end{aligned} \tag{3.78}$$

where $\gamma$ is the average SNR of the source-destination link, $\gamma = d^{-\alpha}\ E_s/N_0$.

In SPNC scheme, 3 times slots are used to convey 2 blocks of information (1 block for each user). Hence the rate for SPNC scheme is same as network coding based MARC which is $\frac{3R}{2}$. In order to efficiently calculate the outage probability for SPNC scheme, outage probability for $S_1$ and $S_2$ can be expressed as

$$P_{out,i} = \sum_{j=1}^{6} P_{out,i,\mathbf{Z}_j} \tag{3.79}$$

for i = 1, 2; $P_{out,i}$ represents outage probability for $i^{th}$ source, and $P_{out,i,\mathbf{Z}_j}$ represents the outage event of $i^{th}$ source jointly with event $\mathbf{Z}_j$.

Let $\mathbb{E}_1$, $\mathbb{E}_2$ and $\mathbb{E}_r$ denote the events in which $S_1$-D link, $S_2$-D link and R-D link are in outage, respectively, and $\overline{\mathbb{E}}_1$, $\overline{\mathbb{E}}_2$ and $\overline{\mathbb{E}}_r$ denote the events in which $S_1$-D link, $S_2$-D link and R-D link are *not* in outage, respectively.

Under the events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_4$, $p = m_1$. Therefore $S_1$ is in outage when both $S_1$-D link and R-D link are in outage i.e. the event $\mathbb{E}_1 \cap \mathbb{E}_r$ occur. Hence the outage probability for $S_1$ under the events $\mathbf{Z}_j$, j = {1, 2, 3, 4}, is given by

$$\begin{aligned} P_{out,1,\mathbf{Z}_j}(R) &= P(\mathbb{E}_1 \cap \mathbb{E}_r \cap \mathbf{Z}_j) \\ &= P(x_1 < C, x_r < C, \mathbf{Z}_j) \end{aligned} \tag{3.80}$$

where C = $\frac{2^{(3R/2)}-1}{E_s/N_0}$.

Under the events $\mathbf{Z}_5$ and $\mathbf{Z}_6$, $p = m_1 \oplus m_2$. Therefore $S_1$ is in outage when either of following two events takes place

1. Both $S_1$-D link and R-D link are in outage i.e. the event $\mathbb{E}_1 \cap \mathbb{E}_r$ occur.

2. Both $S_1$-D link and $S_2$-D link are in outage i.e. the event $\mathbb{E}_1 \cap \mathbb{E}_2$ occur.

Therefore the outage probability of $S_1$ under the event $\mathbf{Z}_j$, j = {5, 6} is given by

$$\begin{aligned} P_{out,1,\mathbf{Z}_j} &= P\left(((\mathbb{E}_1 \cap \mathbb{E}_r) \cup (\mathbb{E}_1 \cap \mathbb{E}_2)) \cap \mathbf{Z}_j\right) \\ &= P(\mathbb{E}_1 \cap \mathbb{E}_r \cap \mathbf{Z}_j) + P(\mathbb{E}_1 \cap \mathbb{E}_2 \cap \mathbf{Z}_j) - P(\mathbb{E}_1 \cap \mathbb{E}_2 \cap \mathbb{E}_r \cap \mathbf{Z}_j) \\ &= P(\mathbb{E}_1 \cap \mathbb{E}_r \cap \mathbf{Z}_j) + P(\mathbb{E}_1 \cap \mathbb{E}_2 \cap \overline{\mathbb{E}}_r \cap \mathbf{Z}_j) \end{aligned} \tag{3.81}$$

Hence, the outage probability for $S_1$ for events $\mathbf{Z}_j$, j = {5, 6} is given by

$$P_{out,1,\mathbf{Z}_j}(R) = P(x_1 < C, x_r < C, \mathbf{Z}_j) + P\left(x_1 < C, x_2 < \frac{C}{\mu}, x_r > C, \mathbf{Z}_j\right) \tag{3.82}$$

These probabilities can be calculated by integrating the joint pdf of $x_1$, $x_2$ and $x_r$, $f(\underline{\mathbf{x}}) = \mu d_1^\alpha d_2^\alpha d_r^\alpha \; e^{-(d_1^\alpha x_1 + \mu d_2^\alpha \cdot x_2 + d_r^\alpha x_r)}$ using appropriate limits specified by $\mathbf{Z}_j$. After integrating and simplifying, the total outage probability for $S_1$ can be calculated using Eq.(3.79) and can be expressed as

$$P_{out,1} = 1 - e^{-C_1} - \left(\frac{d_1^\alpha}{d_1^\alpha + \mu d_2^\alpha}\right) e^{-C_r} + \left(\frac{d_1^\alpha}{d_1^\alpha + \mu d_2^\alpha}\right) e^{-(C_1 + \mu C_2 + C_r)} \tag{3.83}$$

where $C_i = C \cdot d_i^\alpha = \frac{2^{(3R/2)}-1}{d_i^{-\alpha} E_s/N_0}$, {i = 1, 2, r } and we assume that $\mu < 1$.

Next we consider the outage for source $S_2$. Under the events $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$ and $\mathbf{Z}_4$, $p = m_1$. Therefore $S_2$ is in outage when the direct link $S_2$-D is in outage. Hence the outage probability for $S_2$ under the events $\mathbf{Z}_j$, j = {1, 2, 3, 4} is given by

$$\begin{aligned} P_{out,2,\mathbf{Z}_j} &= P(\mathbb{E}_2 \cap \mathbf{Z}_j) \\ &= P\left(x_2 < \frac{C}{\mu}, \mathbf{Z}_j\right) \end{aligned} \tag{3.84}$$

Under the events $\mathbf{Z}_5$ and $\mathbf{Z}_6$, $p = m_1 \oplus m_2$. Therefore $S_2$ is in outage when either of following two events takes place

1. Both $S_2$-D link and R-D link are in outage i.e. the event $\mathbb{E}_2 \cap \mathbb{E}_r$ occur.

2. Both $S_2$-D link and $S_1$-D link are in outage i.e. the event $\mathbb{E}_2 \cap \mathbb{E}_1$ occur.

The outage probability of $S_2$ under the event $\mathbf{Z}_j$, j = {5, 6} can be simplified in similar manner

$$P_{out,2} = 1 - \frac{\mu d_1^\alpha d_2^\alpha}{(d_1^\alpha + d_r^\alpha)(d_1^\alpha + \mu d_2^\alpha + d_r^\alpha)} - e^{-C_2} - \left(\frac{d_1^\alpha + 2d_r^\alpha}{d_1^\alpha + d_r^\alpha}\right) e^{-(C_1+C_r)} + \left(\frac{\mu d_2^\alpha}{d_1^\alpha + \mu d_2^\alpha}\right) e^{-C_r} + \left(\frac{d_1^\alpha}{d_1^\alpha + \mu d_2^\alpha} + \frac{d_r^\alpha}{d_1^\alpha + \mu d_2^\alpha + d_r^\alpha}\right) e^{-(C_1+\mu C_2+C_r)} + \left(\frac{\mu d_2^\alpha}{d_1^\alpha + \mu d_2^\alpha + d_r^\alpha}\right) e^{-\frac{1}{\mu}(C_1+\mu C_2+C_r)} \tag{3.87}$$

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as was done for $S_1$ in Eq.(3.81) and can be expressed as

$$\begin{aligned} P_{out,2,\mathbf{Z}_j} &= P(((\mathbb{E}_2 \cap \mathbb{E}_r) \cup (\mathbb{E}_2 \cap \mathbb{E}_1)) \cap \mathbf{Z}_j) \\ &= P(\mathbb{E}_2 \cap \mathbb{E}_r \cap \mathbf{Z}_j) + P(\mathbb{E}_2 \cap \mathbb{E}_1 \cap \mathbf{Z}_j) - P(\mathbb{E}_1 \cap \mathbb{E}_2 \cap \mathbb{E}_r \cap \mathbf{Z}_j) \\ &= P(\mathbb{E}_2 \cap \mathbb{E}_r \cap \mathbf{Z}_j) + P(\mathbb{E}_2 \cap \mathbb{E}_1 \cap \overline{\mathbb{E}}_r \cap \mathbf{Z}_j) \end{aligned} \tag{3.85}$$

Hence the outage probability for $S_2$ for events $\mathbf{Z}_j$, j = $\{5, 6\}$ is given by

$$P_{out,2,\mathbf{Z}_j} = P\left(x_2 < \frac{C}{\mu}, x_r < C, \mathbf{Z}_j\right) + P\left(x_2 < \frac{C}{\mu}, x_1 < C, x_r > C, \mathbf{Z}_j\right) \tag{3.86}$$

These probabilities can be calculated by integrating the joint pdf $f(\underline{\mathbf{x}})$ using appropriate limits specified by $\mathbf{Z}_j$. After integrating and simplifying, the total outage probability for $S_2$ can be calculated using Eq.(3.79) and can be expressed as Eq.(3.87) assuming $\mu < 1$.

# CHAPTER 4. NUMERICAL RESULTS

Now we present the numerical results. We assume that the two sources $S_1$ and $S_2$ both are at the distance of 1 from the destination D and the relay node R at distance of 0.5 from D. The path loss exponent is assumed to be $\alpha = 4$. The transmit SNRs of all the sources and relay are assumed to be the same. Without loss of generality, we assume that $S_1$ has a higher priority than $S_2$.

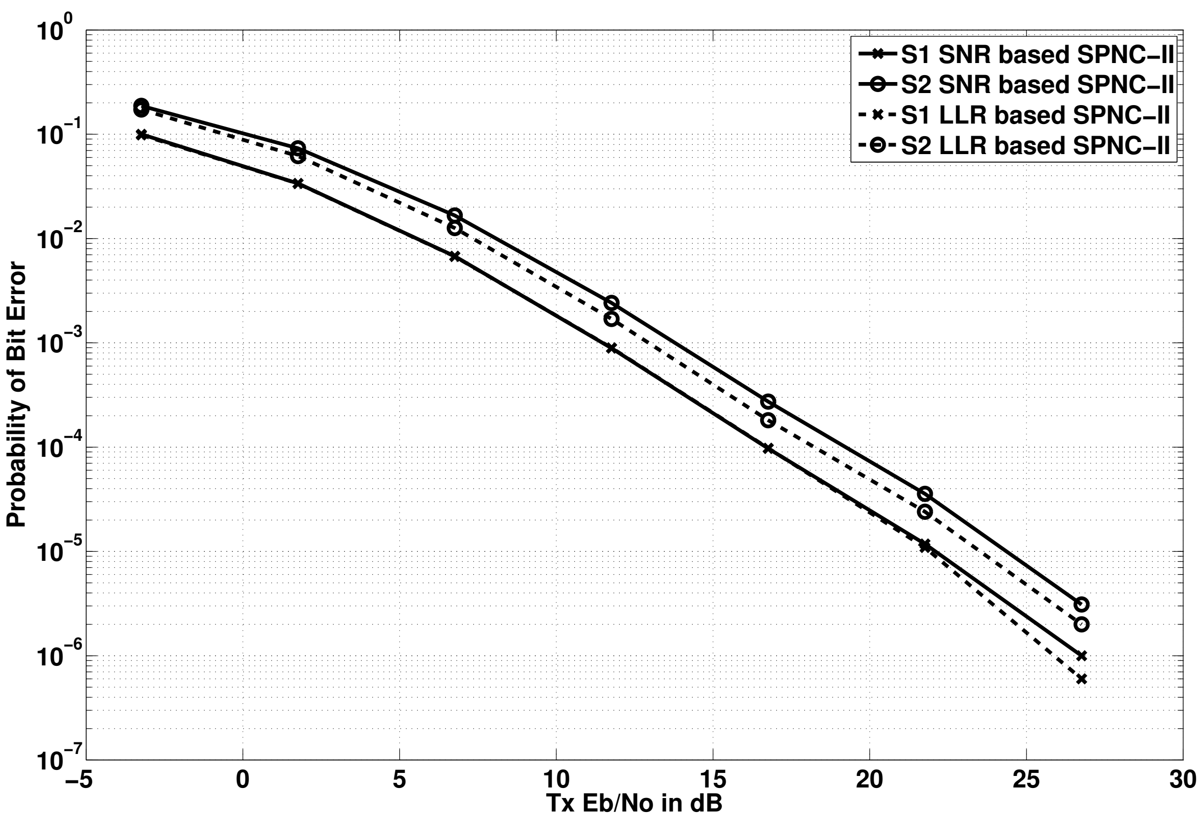


Figure 4.1 Comparison of SNR based and LLR based SPNC-II scheme

Fig. 4.1 compares the bit error probabilities of the SNR-based and the LLR-based SPNC-II scheme. We find that the performance loss of the SNR-based SPNC-II scheme relative to the

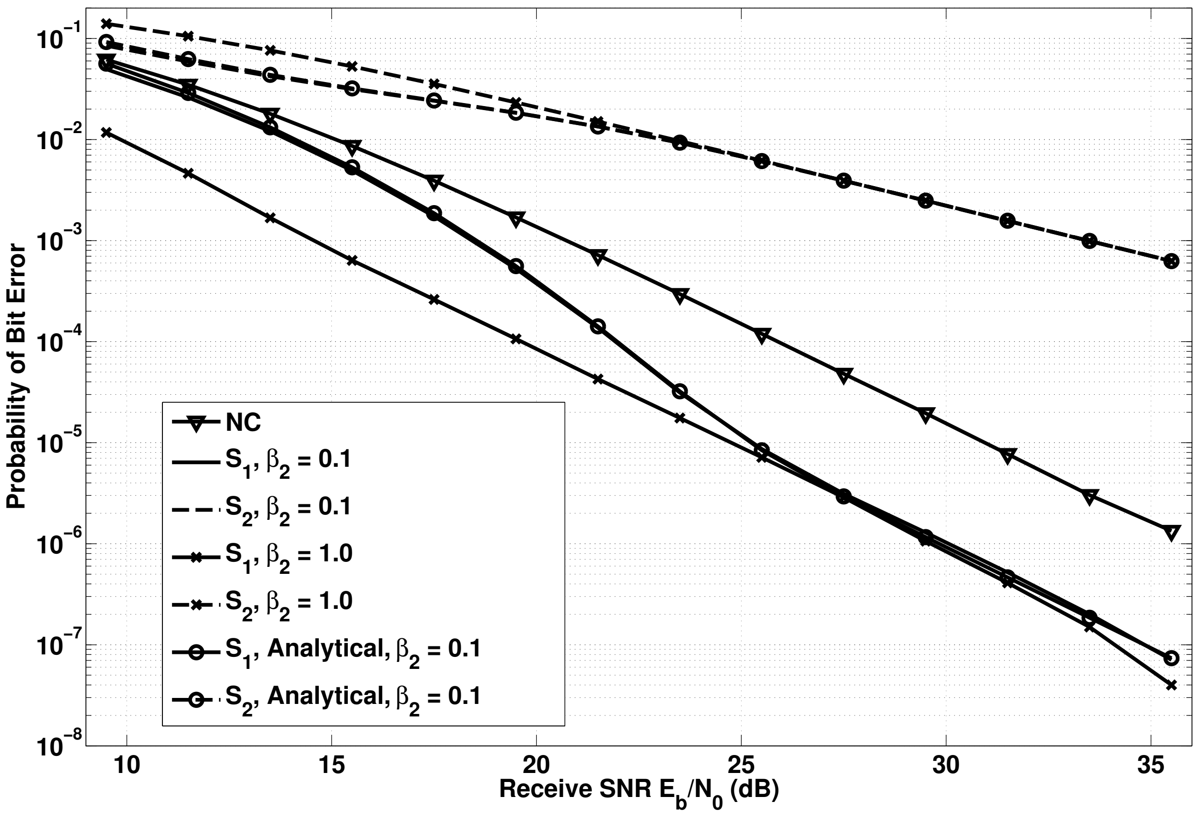


Figure 4.2 Probability of bit error with SNR-based SPNC-I for 2 sources

LLR-based SPNC-II scheme is minor and, therefore, the SNR-based SPNC-II scheme can be effectively used in practice without a significant performance loss.

Fig. 4.2 shows the bit error probability with the SNR-based SPNC-I against the received SNR, $E_b/N_0$, which is given by $E_b/N_0 = (2\gamma_s + \gamma_r)/2$, where $\gamma_s$ and $\gamma_r$ are the received SNRs at the destination from source and relay, respectively and perfect channel estimation. We can see that the bit error probability of prioritized source $S_1$ decreases with increasing threshold $\beta_2$, while that of non-prioritized source $S_2$ increases. When $\beta_2 = 1$, the diversity order for $S_1$ and $S_2$ is 2 and 1, respectively, which is what the conventional hard prioritized network coding provides. However, at high SNR ($\geq$ 15dB), the diversity order of 2 and 1 for $S_1$ and $S_2$ can be achieved with lower threshold ($\beta_2 = 0.1$). This shows that SPNC-I can provide different levels of reliability for different source nodes by adjusting the threshold. Also shown in the figure is the bit error probability with traditional network coding i.e $p = m_1 \oplus m_2$, which corresponds to the special case of $\beta_2 = 0$. We also see that the simulation results closely match with the

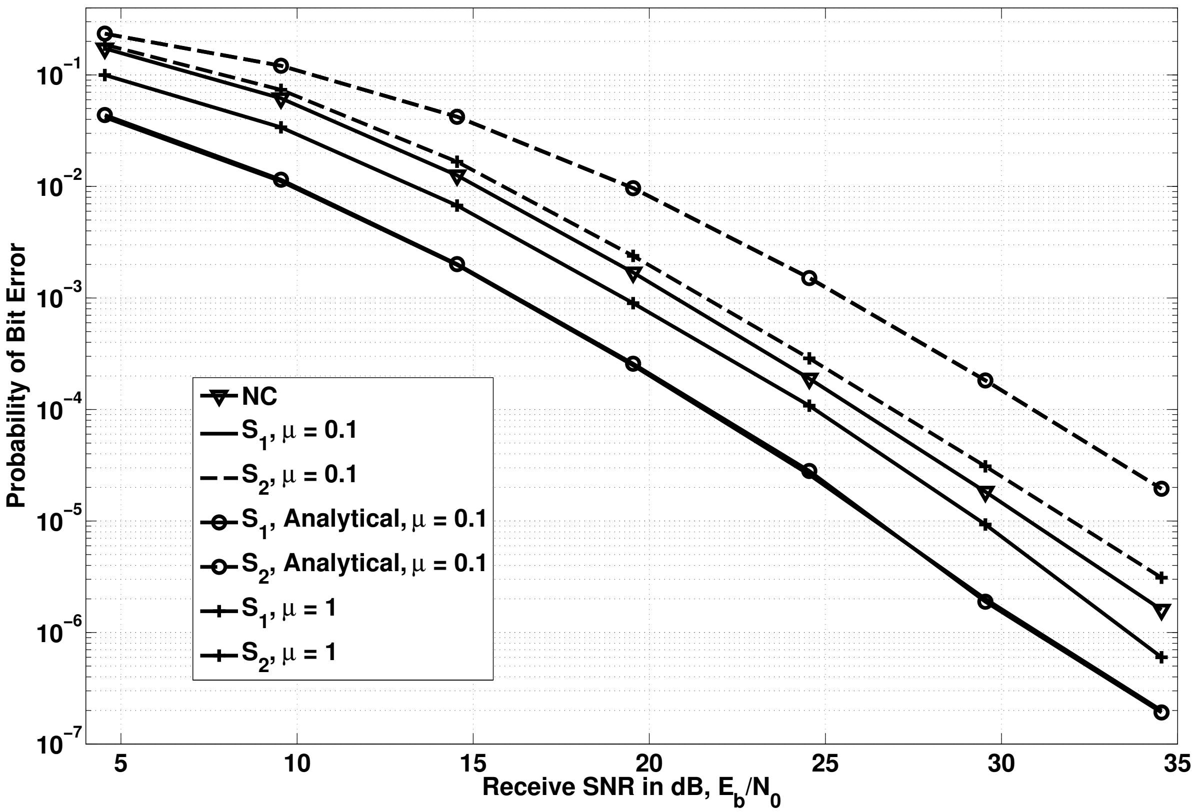


Figure 4.3 Probability of bit error with SPNC-II for 2 sources, $\mu = 0.1,\ 1.0$

theoretical analysis.

Fig. 4.3 shows the bit error probability with SPNC-II against the received SNR, $E_b/N_0$ assuming perfect channel estimation. We can see that the bit error probability of prioritized source $S_1$ increases while that of non-prioritized source $S_2$ decreases with increasing scale-factor $\mu$. Unlike SPNC-I, the diversity order for both $S_1$ and $S_2$ is 2. Hence, with different values of $\mu$, one can achieve different levels of performance without a loss in diversity for either user. This leads to a smaller performance loss for $S_2$ for a given performance gain for $S_1$, which will result in a higher overall throughput. We can also see that the theoretical analysis matches quite closely with simulation results.

Fig. 4.4 shows the comparison of the bit error probability of $S_1$ and $S_2$ for SPNC-II scheme evaluated by using union bound and upper bound on Q function, asymptotic bound on bit error probability (Eqs.(3.66), (3.69)) and their respective union bound on bit error probability for $\mu = 0.1$. As shown in the figure, the bounds are tight and is within 2 dB of exact bit error

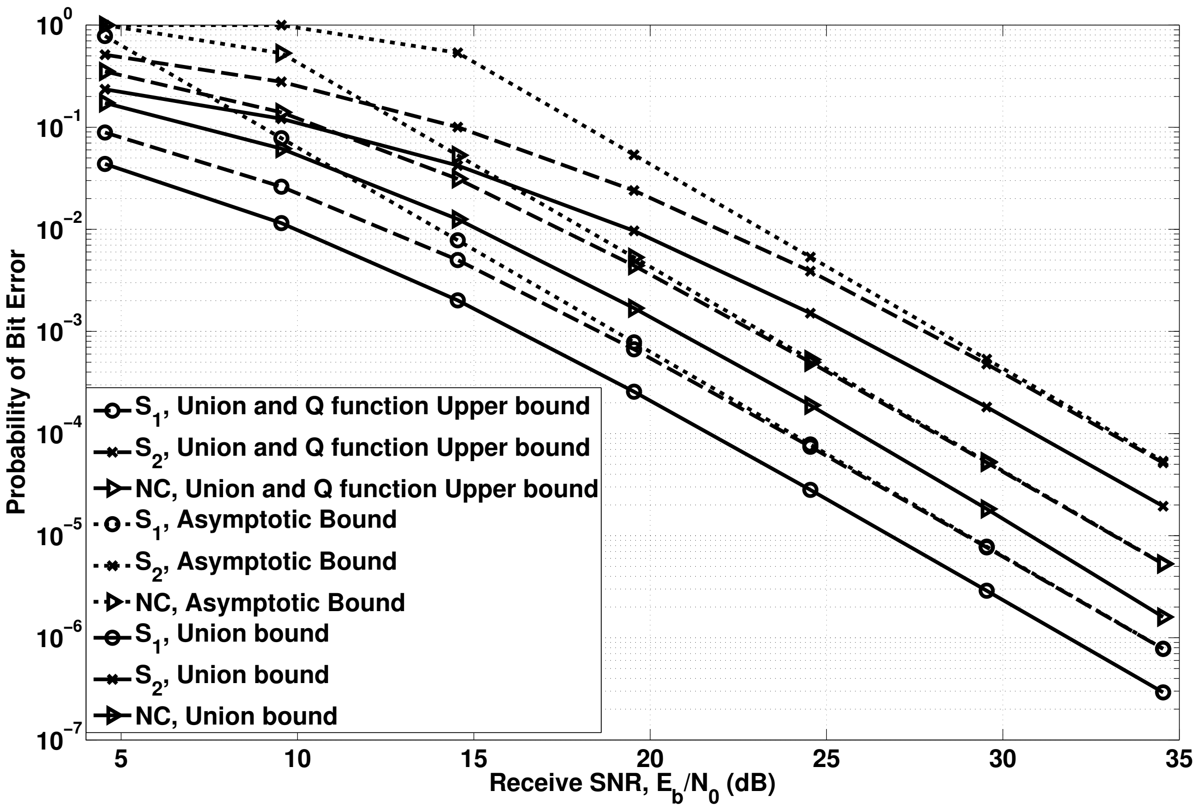


Figure 4.4 SPNC-II asymptotic bounds for $\mu = 0.1$

probability.

Fig. 4.5 shows the probability of bit error for $S_1$ and $S_2$ with respect to the scale-factor, $\mu$. As expected, the error probability for prioritized source $S_1$ increases while that of non-prioritized source $S_2$ decreases with increasing $\mu$ and they converge to the same value at very large $\mu$.

Fig. 4.6 shows the probability of bit error for SPNC-II versus the received SNR, $E_b/N_0$, with channel estimation error. It does introduces small performance loss because of estimation error. However there is no loss in diversity order of the 2 sources and hence SPNC-II scheme can be used effectively for prioritization in practice.

Fig. 4.7 shows the probability of bit error with SPNC-II versus normalized Doppler frequency, $\omega_{ND}$, and $E_{ps,1} = d_1^{-\alpha} E_1$, $E_{ps,2} = d_2^{-\alpha} E_2$, $E_{ps,r} = d_r^{-\alpha} E_r$. The normalized Doppler frequency affects the channel estimation error variance. As the Doppler frequency increases, the channel changes faster and thus the estimation error variance increases. It follows from

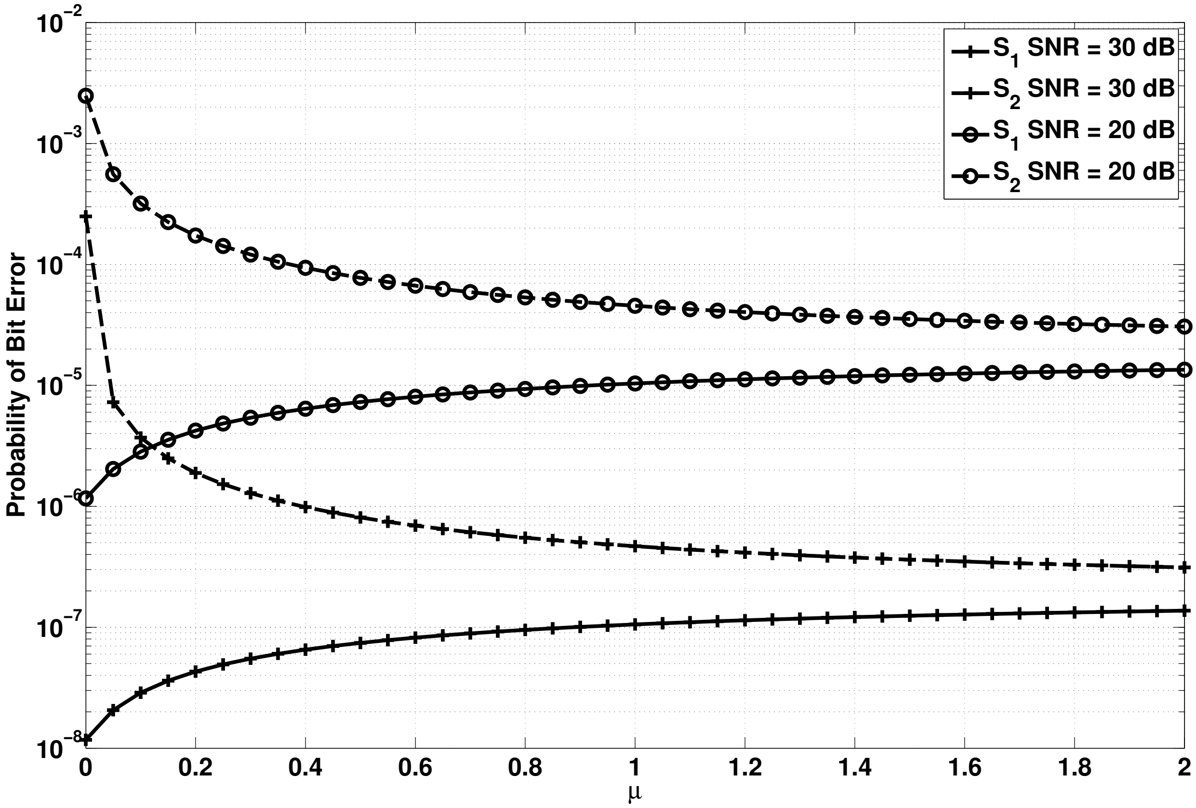


Figure 4.5 Probability of bit error versus scale factor ($\mu$) for SPNC-II

Eq.(3.76) that the increase of channel estimation error variance results in a lower effective SNR. We can see that the probability of bit error increases with increasing Doppler frequency for both prioritized and non-prioritized source nodes at the same rate.

Fig. 4.8 shows the outage probability for $S_1$ and $S_2$ using SPNC-II scheme and $\mu = 0.1$. We can see that both the sources have diversity order two. We also see that the simulation results closely match with the theoretical analysis.

Fig. 4.9 shows the outage probability for SPNC-II scheme versus Rate (R) for $\mu = 0.1$ and SNR $=$ 20dB. As expected, the prioritized user has lowest outage probability. An interesting observation is the improvement in outage probability for prioritized user $S_1$ is more than the loss for non-prioritized user $S_2$ with respect to network coding. This leads to overall rate gain for the system compared to network coding case. This is because whenever the channel from $S_2$ to D is sufficiently strong, the relay forwards the data only from $S_1$ to destination. This leads to a significant rate increase for $S_1$ especially in the cases when the channel from $S_1$ to

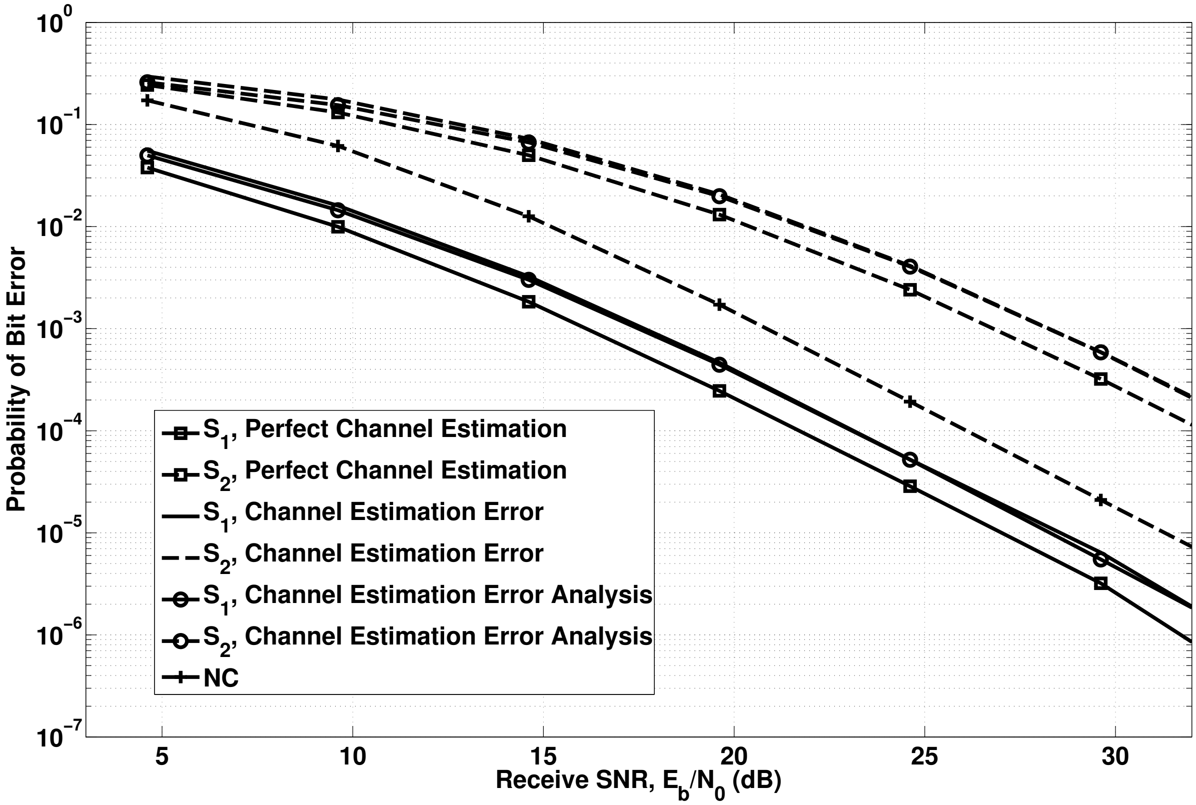


Figure 4.6 Probability of bit error for SPNC-II with channel estimation error, $\mu = 0.1$, L = 10, $\omega_{ND} = 0.04\pi$.

D is weak. At the same time, since the channel from $S_2$ to D is sufficiently strong, the loss in rate for $S_2$ is quite less compared to network coding. Hence there is a overall rate gain for the system by using SPNC.

Fig. 4.10 shows the outage probability for modified SPNC-II scheme versus Receive SNR for $\mu = 0.1$. We modify the SPNC-II as follows:

$$p = \begin{cases} m_1 \oplus m_2 & \text{if } g_2 < \mu \cdot \min\{g_1, g_r\} \text{ and } \min\{g_1, g_r\} > C \\ m_1 & \text{if } g_2 \geq \mu \cdot \min\{g_1, g_r\} \end{cases} \tag{4.1}$$

where C $= \frac{2^R - 1}{E_s/N_0}$. The condition $\min\{g_1, g_r\} > C$ ensures that both the direct path from $S_1$-D and R-D are not in outage, only then the relay transmits network coded bit; else if any one of afore mentioned paths are in outage, the relay transmits data received from prioritized source $S_1$. Simulation results show a little performance improve, about 0.5 dB, for $S_1$ at high SNR and doesn't shows any performance improvement or loss for $S_2$. This shows that SPNC-II scheme

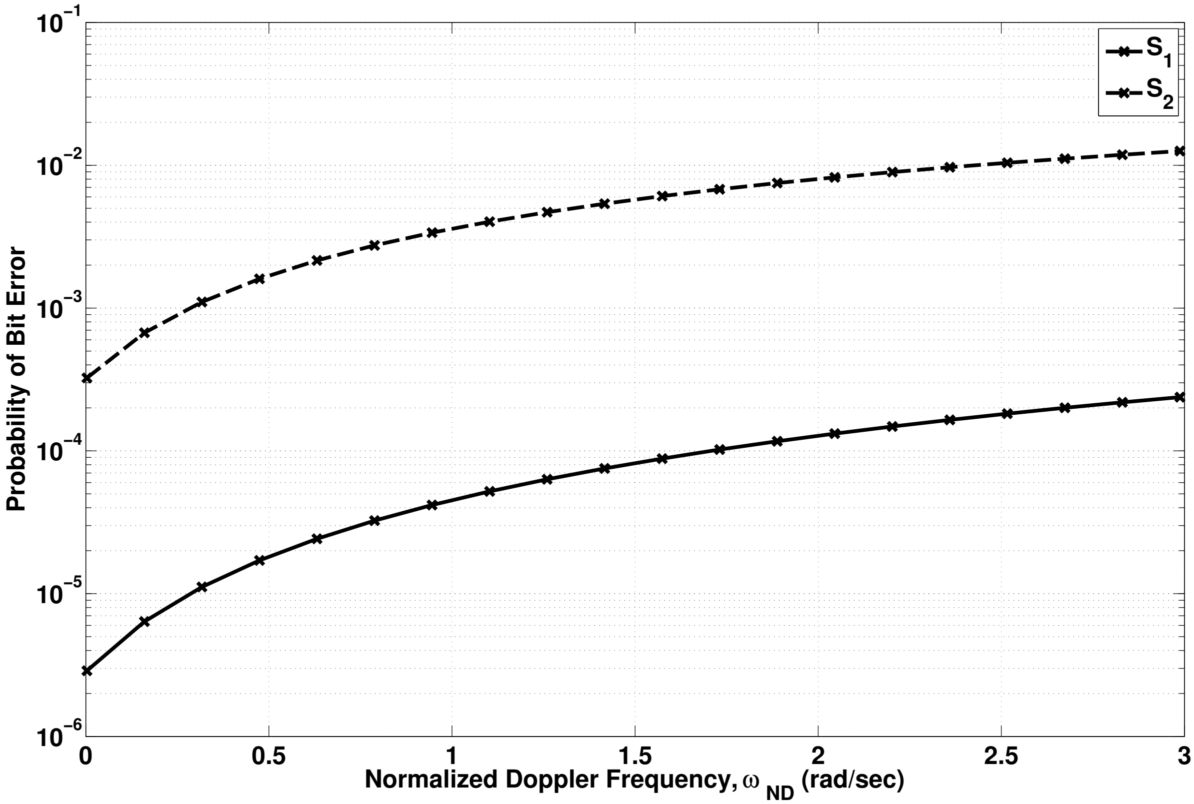


Figure 4.7 Probability of bit error for SPNC-II versus normalized Doppler frequency ($\omega_{ND}$), $E_b/N_0$ = 20 dB, L = 10.

is good scheme and only needs judicious choice of $\mu$.

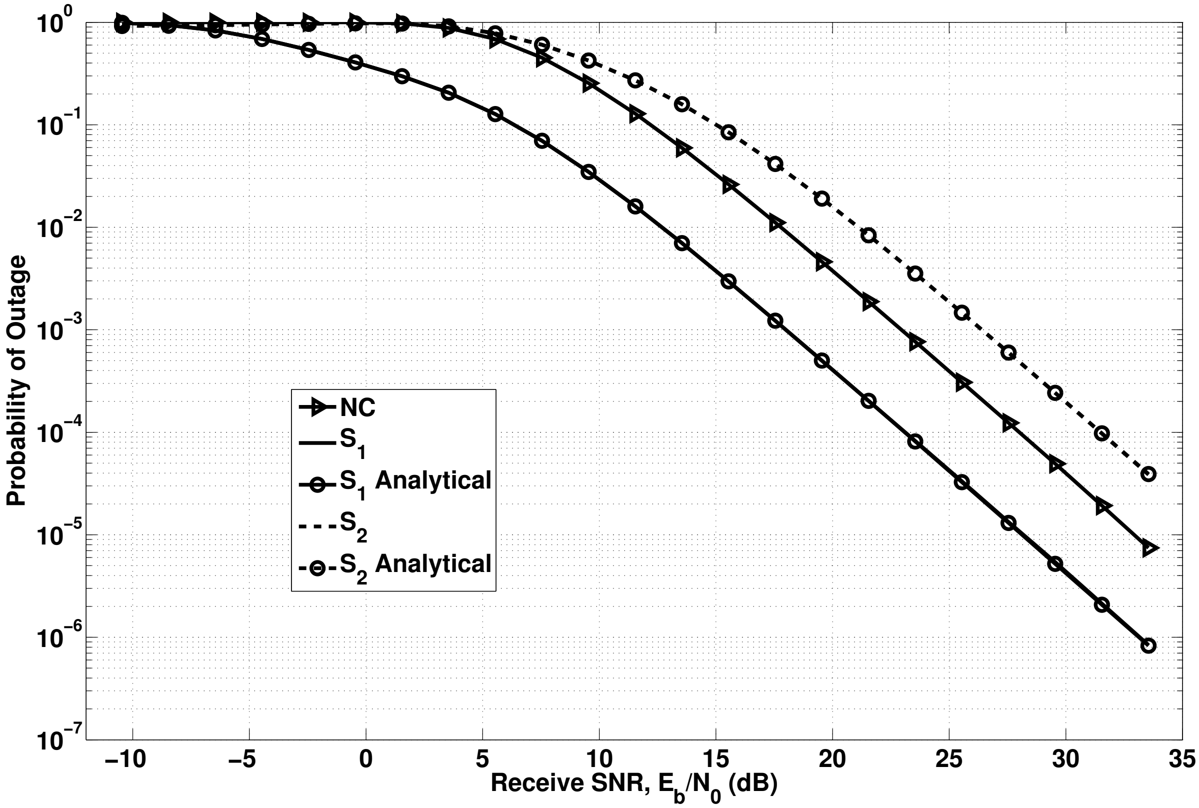


Figure 4.8 SPNC-II Outage Probability Vs Receive SNR, $\mu = 0.1$,

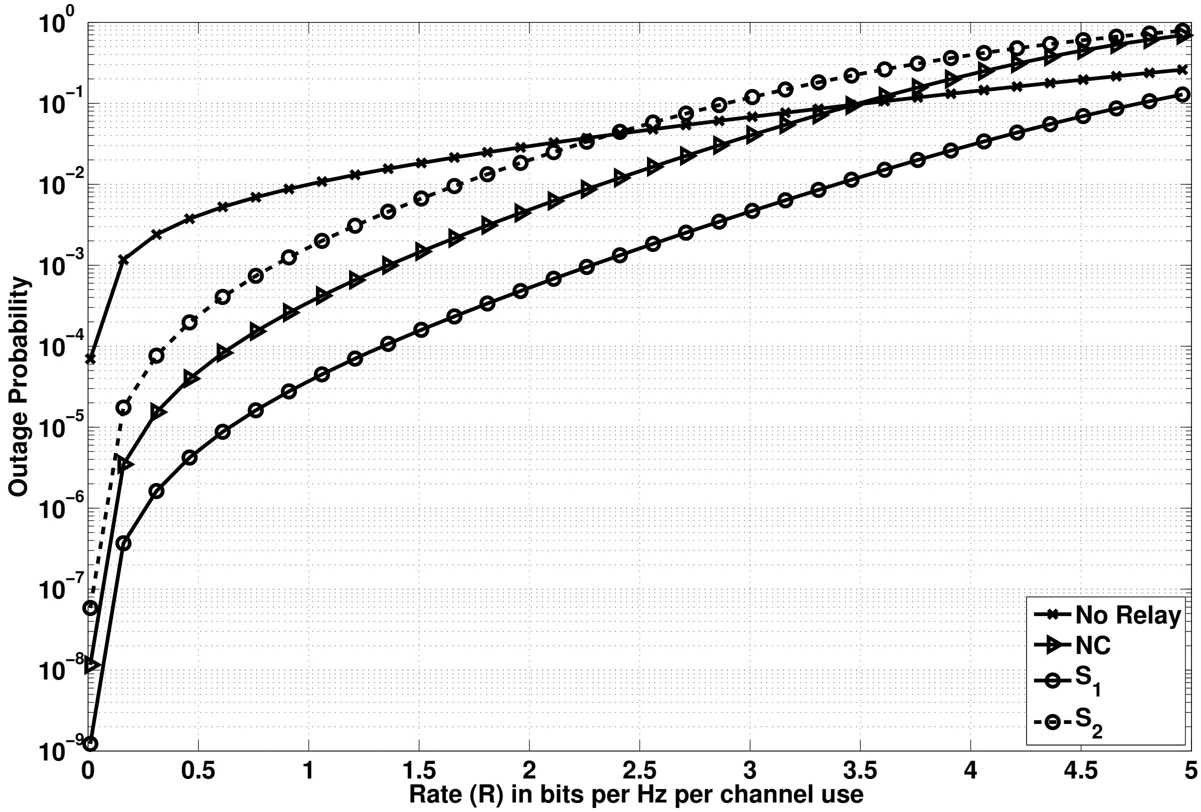


Figure 4.9 SPNC-II Outage Probability Vs Rate, $\mu = 0.1$, Transmit SNR = 20dB

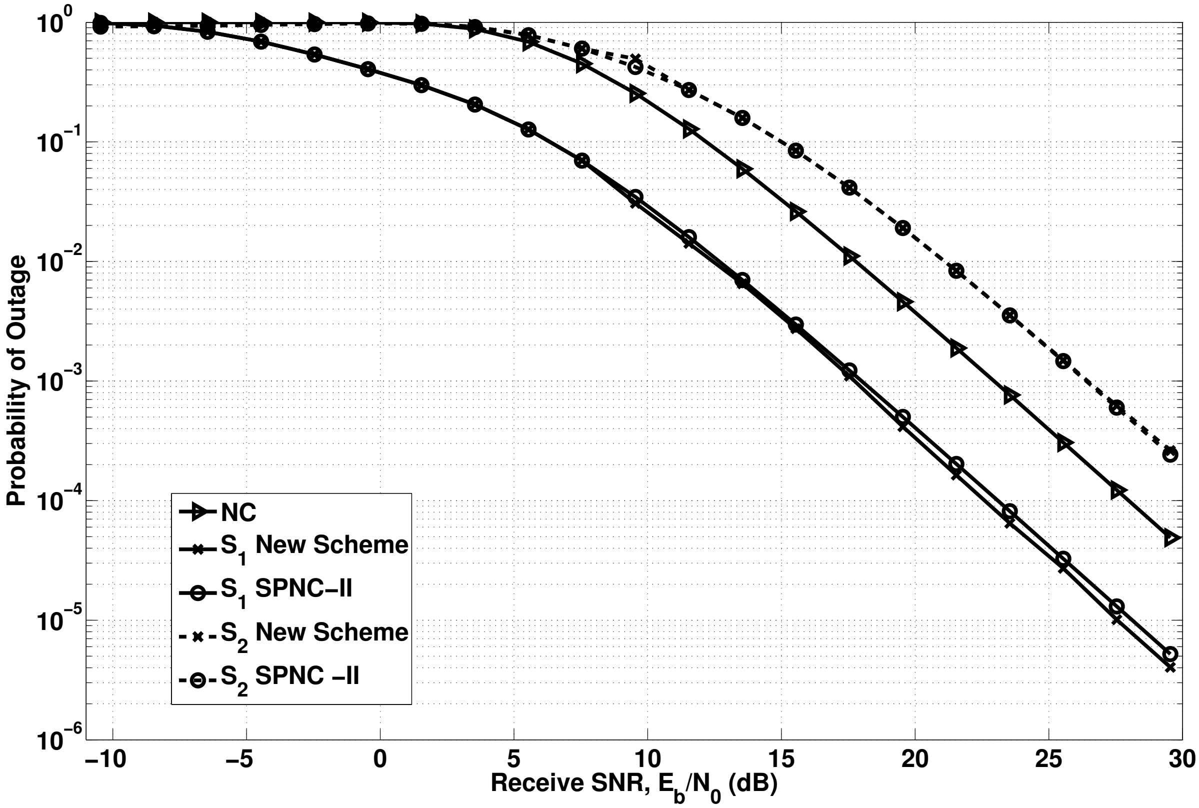


Figure 4.10 Modified SPNC-II Outage Probability Vs Receive SNR, $\mu = 0.1$

# CHAPTER 5. CONCLUSION

We proposed soft prioritized network coding techniques that enable a soft-level prioritized service to different nodes in wireless multiple access relay networks. The proposed techniques utilize the channel state information of source-to-destination links in determining the network encoding rule at the relay. We showed that the proposed techniques can provide soft-level prioritized services, ranging from no diversity to full diversity, depending on the assistance needs. Given that we expect a growing need for variable user-specific service, the proposed techniques can provide a user tailored service that brings fine-tuned user satisfaction. The future work is to extend the scheme to multi-source, multi-relay network and using higher order modulation schemes.

# CHAPTER 6. FUTURE WORK

- Generalize the scheme to multisource, multirelay scenario.
- Generalize the scheme for errors between sources and relay.
- Evaluate the SPNC scheme along with some channel coding scheme.

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